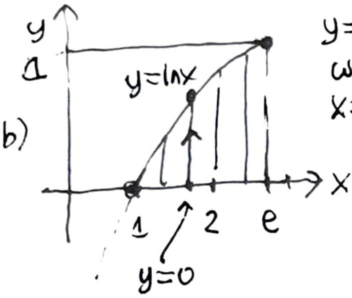
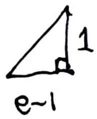
MAT2500-01/02 Q25 Takehome Test 3 Answers (1)

① $A = \int_{x=1}^{x=e}\int_{y=0}^{y=\ln x} 1\,dy\,dx \overset{\text{Maple}}{=} \boxed{1}$

b)



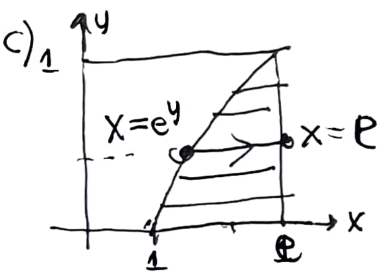
y=0..ln x
while
x=1..e



Area $= \frac{1}{2}(1)(e-1)$

$\approx 0.86 \sim 1$ yes ✓
should be a bit less
(ln x concave down)

c)



$x=e^y..1$
while
$y=0..1$

$y=\ln x \to e^y = e^{\ln x} = x$

d) $A = \int_0^1 \int_{e^y}^{e} 1\,dx\,dy$

e) $= \int_0^1 x \Big|_{x=e^y}^{x=e} dy$

$= \int_0^1 e - e^y\,dy = ey - e^y \Big|_0^1$

$= \cancel{e} - e - (0-1) = \boxed{1}$ ✓

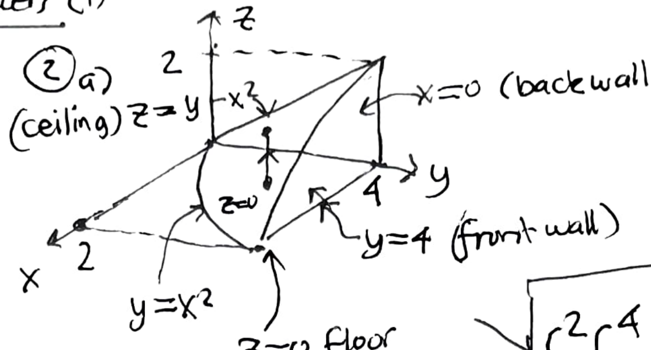
same!

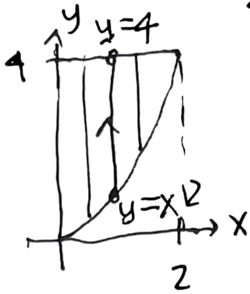
(original order requires integration by parts!)

② a) $x=0,\ y=4,\ z=0,\ z=y-x^2=0 \to y=x^2$

intersections $\begin{cases} z=4-x^2 \leftarrow 4 \\ z=y \leftarrow x=0 \end{cases}$

② a)

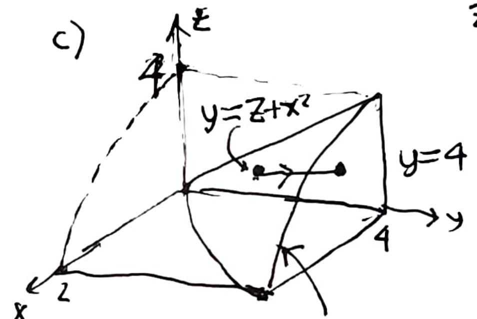




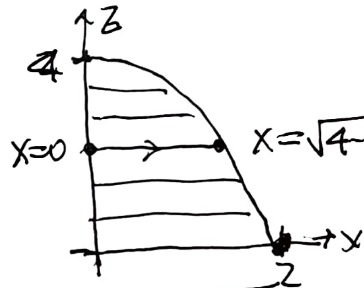
b) $\boxed{\int_0^2 \int_{x^2}^4 \int_0^{y-x^2} y\,dz\,dy\,dx}$

$= \boxed{\frac{512}{21}} \approx 24.381$

c)



$z=y-x^2 \to y=z+x^2$

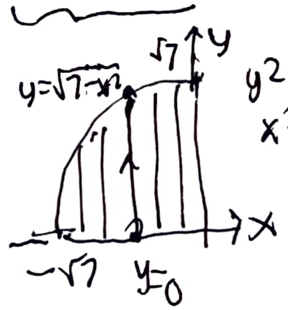


$x=0..\sqrt{4-z}$
while $z=0..4$

$y=4 \wedge y=z+x^2$
$4=z+x^2$
$z=4-x^2$
$x=\sqrt{4-z}$

$\boxed{\int_0^4 \int_0^{\sqrt{4-z}} \int_{z+x^2}^{4} y\,dy\,dx\,dz} = \boxed{\frac{512}{21}}$ ✓

③ a) $\int_{x=-\sqrt{7}}^{x=0} \int_{y=0}^{y=\sqrt{7-x^2}} \int_{z=\frac{3}{\sqrt{7}}\sqrt{x^2+y^2}}^{z=\sqrt{16-x^2-y^2}} z\,dz\,dy\,dx$



$y^2=7-x^2$
$x^2+y^2=7$
circle

$\begin{cases} z^2=16-x^2-y^2 \\ x^2+y^2+z^2=16=4^2 \text{ sphere} \end{cases}$

$z^2 = \frac{9}{7}(x^2+y^2)$ cone

$(x^2+y^2)\underbrace{\left(1+\frac{9}{7}\right)}_{\frac{16}{7}} = 16$

intersection $x^2+y^2=7$ circle: $r=\sqrt{7}$

$z=\frac{3}{\sqrt{7}}(\sqrt{7})=3$

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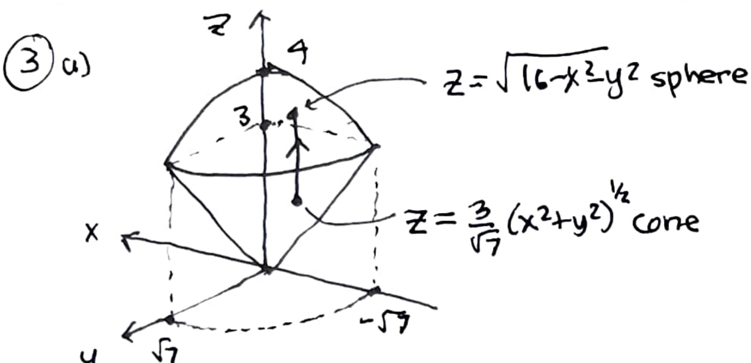
③ a)

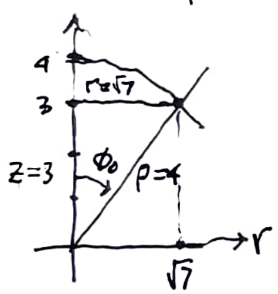


"quarter snow cone"

b) sphere: $\underbrace{x^2+y^2}+z^2=16$ cone: $z=\frac{3}{\sqrt7}r$

c) $\downarrow$ $r^2+z^2=16 \quad \Leftrightarrow \quad z=\sqrt{16-r^2}$

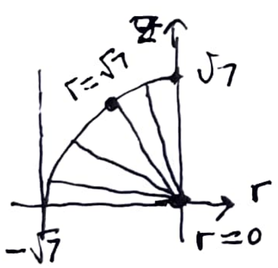
$\downarrow$ $\rho^2=16 \to \rho=4$



$\boxed{\phi_0=\arccos 3/4}$ ← simplest?

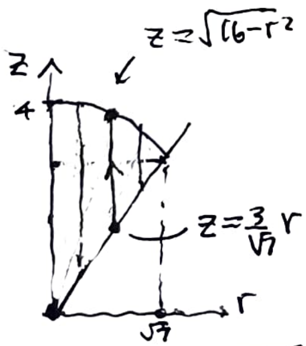
$=\arctan\sqrt7/3$

$=\arcsin\sqrt7/4$



$r=$

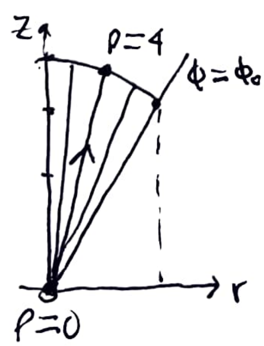
while $\theta=\frac{\pi}{2}..\pi$



$z=\frac{3}{\sqrt7}r..\sqrt{16-r^2}$

while $r=0..\sqrt7$

$\rho=0..4$
while
$\phi=0..\phi_0$



$$\int_{\frac{\pi}{2}}^{\pi}\int_0^{\sqrt7}\int_{\frac{3}{\sqrt7}r}^{\sqrt{16-r^2}} z(r)\,dz\,dr\,d\theta = \int_{\frac{\pi}{2}}^{\pi}\int_0^{\phi_0}\int_0^4 (\rho\cos\phi)(\rho^2\sin\phi)\,d\rho\,d\phi\,d\theta$$

$\underset{\text{Maple}}{=} \boxed{7\pi \approx 21.991}$

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4 a) sphere: $x^2+y^2+(z-2)^2=4$

$r^2+(z-2)^2=4 \to r^2+z^2-4z+\cancel{4}=\cancel{4} \to r^2+z^2=4z$ → $r=\sqrt{4z-z^2}$

$z^2-4z+r^2=0$ → $z=\frac{4\pm\sqrt{16-4r^2}}{2} = 2\pm\sqrt{4-r^2}$

$\rho^2-4(\rho\cos\phi)=0$

$\rho=4\cos\phi$

plane: $z=3=\rho\cos\phi$

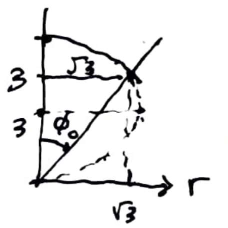
$\rho=3\sec\phi$

$\boxed{\text{intersection}}$ $z=3$, $r^2+z^2=4z$

$r^2+9=12$

$r^2=3$, $r=\sqrt{3}$

$\boxed{(r,z)=(\sqrt{3},3)}$

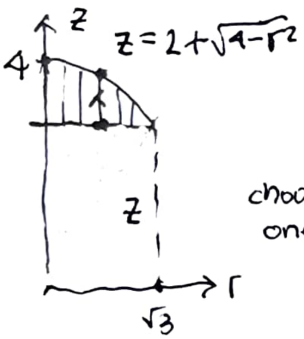


$\tan\phi_0=\frac{\sqrt{3}}{3}=\frac{1}{\sqrt{3}}$

$\phi_0=\arctan\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}$

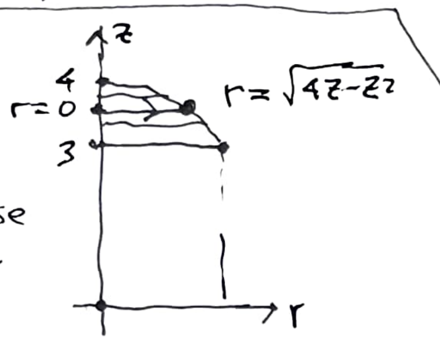
$\rho=4\cos\frac{\pi}{6}=2\sqrt{3}$

$\boxed{(\rho,\phi_0)=(2\sqrt{3},\pi/6)}$

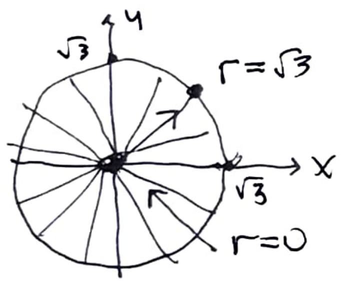


$z=3\,..\,2+\sqrt{4-r^2}$
while $r=0\,..\,\sqrt{3}$

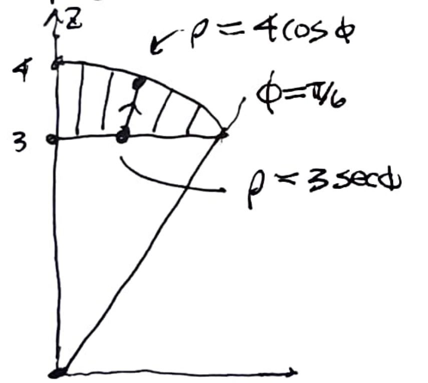
choose one



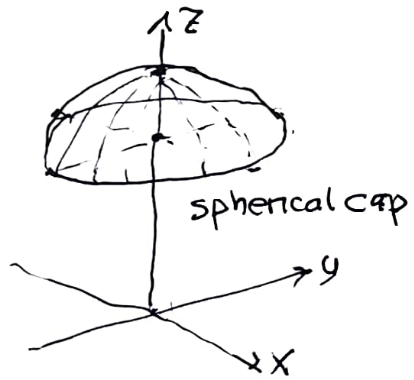
$r=0\,..\,\sqrt{4z-z^2}$
while $z=3\,..\,4$



$r=0\,..\,\sqrt{3}$
while $\theta=0\,..\,2\pi$



$\rho=3\sec\phi\,..\,4\cos\phi$
while $\phi=0\,..\,\pi/6$

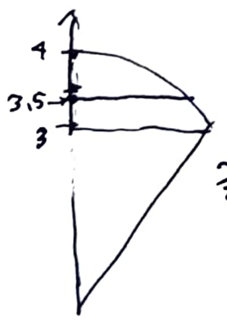


$$\iiint_R f(x,y,z)\,dV=\int_0^{2\pi}\int_0^{\sqrt{3}}\int_3^{2+\sqrt{4-r^2}} f(r\cos\theta,r\sin\theta,z)\,r\,dz\,dr\,d\theta=\int_0^{2\pi}\int_3^4\int_0^{\sqrt{4z-z^2}} f(\dots)\,r\,dr\,dz\,d\theta$$

$$=\int_0^{2\pi}\int_0^{\pi/6}\int_{3\sec\phi}^{4\cos\phi} f(\rho\sin\phi\cos\theta,\rho\sin\phi\sin\theta,\rho\cos\phi)\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$\langle V, M_{yz}, M_{xz}, M_{xy}\rangle=\iiint_R \langle 1,x,y,z\rangle\,dV=\iiint_R\langle 1, r\cos\theta, r\sin\theta, z\rangle\,dV$$

$$=\iiint_R\langle 1,\rho\sin\phi\cos\theta,\rho\sin\phi\sin\theta,\rho\cos\phi\rangle\,dV$$



$\underset{=}{\text{Maple}}$ $\langle \frac{5\pi}{3}, \underbrace{0, 0}_{}, \frac{67\pi}{12}\rangle$ vanish by symmetry, no need to set up integrals

$\bar{z}=\frac{M_{xy}}{V}=\frac{67\pi}{12}\cdot\left(\frac{3}{5\pi}\right)=\boxed{\frac{67}{20}=3.35}$ below midpoint as expected!