

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and **and scan** this test sheet as a cover first page in the PDF scan of your lined paper hand work all on separate sheets.

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: _____

Date: _____

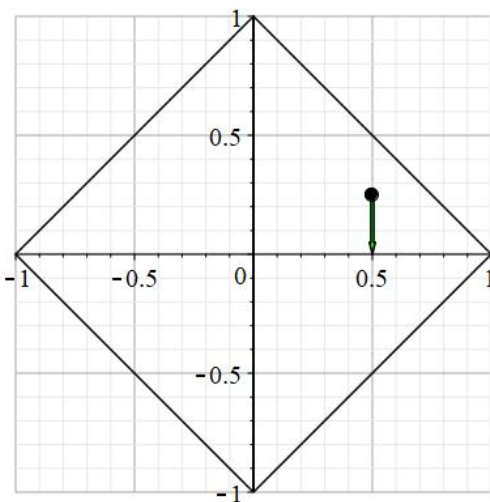
1. Find the dimensions and volume of a rectangular box of maximum volume such that the sum of the lengths of its 8 edges, excluding those 4 horizontal edges at its base, is a constant $P = 12$ ft. You might imagine this to be the aluminum tubing frame of a rectangular tent with 4 sides and a top.

Draw a diagram of the box, introduce your variables and the inequality and equality constraints on their values, evaluate this restricted perimeter and use it to eliminate a variable to obtain a 2 variable maximization problem with clearly stated inequality and equality constraints. Draw a diagram of the allowed region for the 2d max problem in the plane. Solve this problem. Confirm that it is a local maximum using the second derivative test. Then answer the word problem in an English sentence with correct units. Draw a rough diagram of your 3d box with the sides labeled, as a stick figure of only the edges boldly drawn, to appropriate scale.

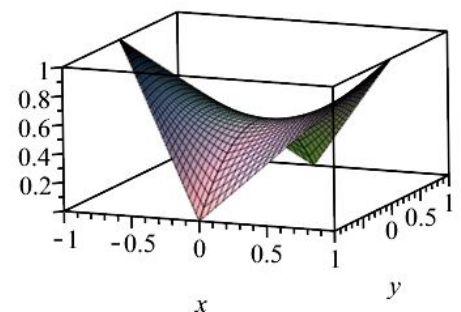
2. The Pavilian has a roof composed of a number of components each in the shape of a hyperbolic paraboloid

$z = \frac{1}{2} (1 + x^2 - y^2)$. Consider a roof segment over the square region of the plane shown in the figure (in

appropriate units), the floor plan at $z = 0$ underneath this part of the roof/ceiling (idealized at zero thickness to this scale)



floor plan



roof

Identify each quantity you evaluate in working this problem with its proper symbol and defining formula.

- a) At the point P on the ceiling above the point $(1/2, 1/4)$ on the floor, a tightly focused spotlight is installed pointing down from the roof along the normal direction. Write the equation for this normal line using a unit normal so that it is parametrized by arclength. What is the center point S of the spot on the floor (point where the normal line intersects the plane $z = 0$) and what is the exact distance from the roof to the floor along this line? How distant is this spot from the point F on the floor directly below the spotlight (exactly and numerically to 4 decimal places)?
- b) If a ball is released on top of the roof at this point, in which direction will it begin rolling down the roof? [Give a horizontal unit vector at the point F on the floor beneath the point on the roof to indicate this direction: think "direction of fastest decrease" of the height function at floor level.]
- c) If a workman walks along the roof above a path from the point F on the floor towards the x -axis at the point $(1/2, 0)$, as shown in the floor plan figure. What is the directional derivative of the height function z of the roof in that direction? What is the approximate change in height if the workman moves to a point 0.02 length units along that direction on the floor below? Does it increase or decrease?

3. A model for the surface area of a human body is given by $S = 0.1091 w^{0.425} h^{0.725}$, where w is the weight (in pounds), h is the height (in inches), and S is measured in square feet.

- a) If the errors in measurement of w and h are at most 2%, use differentials to estimate the maximum percentage error in the calculated surface area. First state the differential of S (simplify!), and the inequalities on the differentials of the two variables, etc. Show every step please.
- b) On some mornings Dr. Bob weighs 160 lbs, and is just short of 72 inches tall (he shrunk since his driver's license height of 6'1"). Evaluate his surface area (units!), and the new surface area if he were to shrink 2 percent in both weight and height, and evaluate the exact percentage difference to 5 decimal places and compare with your differential approximation for the error range. Does it fit in the computed error range?