MAT2500-01/02 20S Test 1 Answers

a) $\vec{r} = \langle 4t^{1/2}, t, t^2 \rangle \quad 0 \le t \le 3/2$ $\qquad \vec{r}(1) = \langle 4,1,1 \rangle$

$\vec{r}\,' = \langle 2t^{-1/2}, 1, 2t \rangle = \vec{v}$ $\qquad \vec{r}\,'(1) = \langle 2,1,2 \rangle$

$\vec{r}\,'' = \langle -t^{-3/2}, 0, 2 \rangle = \vec{a}$ $\qquad \vec{r}\,''(1) = \langle -1,0,2 \rangle$

$|\vec{r}\,'| = \sqrt{\cancel{4}4/t + 1 + 4t^2} = v$ $\qquad |\vec{r}\,'(1)| = \sqrt{4+1+4} = 3$

$\hat{T} = \dfrac{\vec{r}\,'}{|\vec{r}\,'|} = \dfrac{\langle 2t^{-1/2}, 1, 2t \rangle}{\sqrt{4/t + 1 + 4t^2}}$ $\qquad \hat{T}(1) = \frac{1}{3}\langle 2,1,2 \rangle$

$|\vec{r}\,''(1)| = \sqrt{1+4} = \sqrt{5}$

$|\vec{r}\,''| = \sqrt{t^{-3} + 4}$

b) $\vec{r} = r_0 + t\vec{a} = \langle 4,1,1 \rangle + t\langle 2,1,2 \rangle$

$\vec{r} = \langle x y z \rangle$, $r_0 = r(1)$, $\vec{a} = \vec{r}\,'(1)$

$\boxed{\langle x,y,z \rangle = \langle 4+2t, 1+t, 1+2t \rangle}$

c) $\vec{b}(1) = \vec{r}\,'(1) \times \vec{r}\,''(1) = \langle 2,1,2 \rangle \times \langle -1,0,2 \rangle \overset{\text{Maple}}{=} \boxed{\langle 2,-6,1 \rangle}$

$\vec{b}(t) = \langle 2t^{-1/2}, 1, 2t \rangle \times \langle -t^{-3/2}, 0, 2 \rangle \overset{\text{Maple}}{=} \boxed{\langle 2, -6t^{-1/2}, t^{-3/2} \rangle}$

$|\vec{b}(1)| = \sqrt{4+36+1} = \sqrt{41}$

$\hat{B}(1) = \dfrac{\vec{b}(1)}{|\vec{b}(1)|} = \boxed{\dfrac{\langle 2,-6,+1 \rangle}{\sqrt{41}}}$

d) $\hat{N}(1) = \hat{B}(1) \times \hat{T}(1) = \dfrac{\langle 2,-6,1 \rangle}{\sqrt{41}} \times \dfrac{\langle 2,1,2 \rangle}{3} \overset{\text{Maple}}{=} \boxed{\dfrac{\langle -13,-2,14 \rangle}{3\sqrt{41}}}$

e) $0 = \vec{n} \cdot (\vec{r} - \vec{r}_0) = \langle 2,-6,1 \rangle \cdot \langle x-4, y-1, z-1 \rangle$

$\vec{n} = \vec{b}(1)$, $\vec{r}_0 = \vec{r}(1)$

$= 2(x-4) - 6(y-1) + 1(z-1) = 2x - 8 - 6y + 6 + z - 1$

$= 2x - 6y + z - 3$

$\boxed{2x - 6y + z = 3}$

f) $\kappa(t) = \dfrac{|\vec{b}(t)|}{(4/t + 1 + 4t^2)^{3/2}}$ (above)

$\overset{\text{Maple}}{=} \boxed{\dfrac{\sqrt{4 + 36t^{-1} + t^{-3}}}{(4/t + 1 + 4t^2)^{3/2}}} = \dfrac{\sqrt{4t^3 + 36t^2 + 1}}{t^{3/2}\left(\sqrt{(4+t+4t^3)/t}\right)^{3/2}}$

$= \boxed{\dfrac{(4t^3 + 36t^2 + 1)^{1/2}}{(4t^3 + t + 4)^{3/2}} = \dfrac{1}{\rho(t)}}$

$\rho(1) = \dfrac{(4+1+4)^{3/2}}{(4+36+1)^{1/2}} = \boxed{\dfrac{27}{\sqrt{41}}}$

g) $a_T(1) = \hat{T}(1) \cdot \vec{a}(1) = \frac{1}{3}\langle 2,1,2 \rangle \cdot \langle -1,0,2 \rangle$

$= \frac{1}{3}(-2+0+4) = \boxed{\frac{2}{3}}$

$a_N(1) = \hat{N}(1) \cdot \vec{a}(1) = \dfrac{\langle -13,-2,14 \rangle}{3\sqrt{41}} \cdot \langle -1,0,2 \rangle$

$= \dfrac{1}{3\sqrt{41}}(13+0+28) = \dfrac{41}{3\sqrt{41}} = \boxed{\dfrac{\sqrt{41}}{3}}$

h) $L = \int_0^{3/2} |\vec{r}\,'(t)|\,dt = \int_0^{3/2} \sqrt{4/t + 1 + 4t^2}\,dt$

$\overset{\text{Maple}}{\approx} 6.02366 \approx \boxed{6.0237}$

$\vec{r}(3/2) - \vec{r}(0) = \langle 4(3/2)^{1/2}, 3/2, (3/2)^2 \rangle - \vec{0}$

$|\vec{r}(3/2)| = \sqrt{16 \cdot \frac{3}{2} + \frac{9}{4} + \frac{81}{16}} \overset{\text{Maple}}{\approx} 5.60$

curve is a bit longer than the straight line. check ✓

i)

$\vec{C}(1) = \vec{r}(1) + \rho(1)\hat{N}(1)$

$= \langle 4,1,1 \rangle + \dfrac{27}{\sqrt{41}} \dfrac{\langle -13,-2,14 \rangle}{3\sqrt{41}}$

$= \langle 4,1,1 \rangle + \dfrac{9}{41}\langle -13,-2,14 \rangle$

$= \dfrac{\langle 47, 23, 167 \rangle}{41}$ rational numbers!