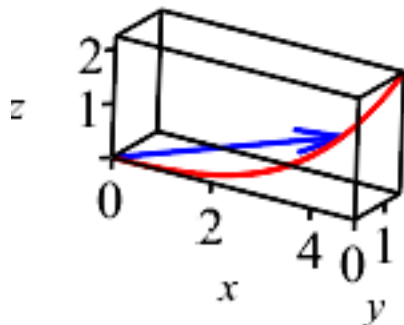


Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

The parametrized curve segment $\vec{r}(t) = \langle 4\sqrt{t}, t, t^2 \rangle, 0 \leq t \leq \frac{3}{2}$.

is shown in the figure together with $\vec{r}(1)$.

a) Evaluate and simplify $\vec{v}(t) = \vec{r}'(t), \vec{a}(t) = \vec{r}''(t), v(t) = |\vec{r}'(t)|$,



$\hat{T}(t), a(t) = |\vec{r}''(t)|$ and their values (including $\vec{r}(t)$) at $t = 1$.

b) Write the parametrized equations of the tangent line through $\vec{r}(1)$.

c) Evaluate and simplify $\vec{b}(t) = \vec{r}'(t) \times \vec{r}''(t)$. Evaluate $\vec{b}(1)$ and $|\vec{b}(1)|$. Then evaluate and simplify the unit vector $\hat{B}(1) = \frac{\vec{b}(1)}{|\vec{b}(1)|}$.

d) Evaluate and simplify the unit normal $\hat{N}(1) = \hat{B}(1) \times \hat{T}(1)$.

e) Write the simplified equation of the osculating plane through $\vec{r}(1)$ containing the tangent vector and the second derivative there.

f) Evaluate the curvature $\kappa(t) = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3}$ and its reciprocal, the radius of curvature $\rho(t)$ and $\rho(1)$.

Simplify $\rho(t)$ to a 2 level fraction.

g) Evaluate the scalar tangential projection $a_T(1)$ along $\hat{T}(1)$ of the acceleration $\vec{a}(1) = \vec{r}''(1)$ and its scalar normal projection $a_N(1) = \hat{N}(1) \cdot \vec{a}(1)$ exactly.

h) Write down an integral formula for the length of the curve $\vec{r}(t)$ for $t=0.. \frac{3}{2}$. Evaluate it numerically to 4 decimal places.

[Optional. Compare with the length of the secant line segment between its endpoints.]

i) **Optional.** The center of the osculating circle has position vector: $\vec{C}(t) = \vec{r}(t) + \rho(t) \hat{N}(t)$. Show that

$$\vec{C}(1) = \left\langle \frac{47}{41}, \dots \right\rangle.$$

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page, with the first test page facing up:

"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: _____

Date: _____

[Note:

> *with(Student[VectorCalculus]) : # needed for dot product to work properly, and magnitude $\|\vec{v}\|$*

> *$r(t) := \langle 4\sqrt{t}, t, t^2 \rangle :$*

> *$r(t); r'(t); r''(t)$*

► **solution**