MAT2705-04/05 20f Quiz 1 Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use **proper mathematical notation**, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not subsitute for them, unless specifically requested.

1. $2\frac{dy}{dx} + y = x,\quad soln:\ y = -2 + x + C\,e^{-\frac{1}{2}x}$

a) Verify that this y satisfies the given differential equation.

b) Find the solution which satisfies the initial condition $y(0) = 3$.

Organize your work as though you were playing professor.

2. Write a differential equation that models the situation:
"The time rate of change of a population P is proportional to the square root of the population."
Explain what sign the constant of proportionality should have and why.
[Use the variable t for time and use d/dt derivative notation.]

[**Optional.** Use Maple to solve your DE with the initial condition $P(0) = P_0$.]

▶ solution

① a) $y = -2 + x + ce^{-x/2}$

$\frac{dy}{dx} = 0 + 1 + ce^{-x/2}(-1/2)$

$= 1 - \frac{c}{2}e^{-x/2}$

$2\frac{dy}{dx} + y = x \rightarrow$

$2(1 - \frac{c}{2}e^{-x/2}) + (-2 + x + ce^{-x/2})$

$= x$

$\cancel{2} - \cancel{ce^{-x/2}} - \cancel{2} + x + \cancel{ce^{-x/2}} = x$

$x = x$ ✓

b) $3 = y(0) = -2 + (0) + ce^{-0/2}$

$= -2 + C \rightarrow \quad C = 3 + 2 = 5$

$$\boxed{y = -2 + x + 5e^{-x/2}}$$

② $\frac{dP}{dt} \propto \sqrt{P} \rightarrow \boxed{\frac{dP}{dt} = k\sqrt{P}},\ P \geq 0$

simple population models usually describe growth

so $\frac{dP}{dt} \geq 0$ would require

$\boxed{k > 0}$

BUT you could argue that the statement is ambiguous and it might describe a decreasing population model

> $P'(t) = k\sqrt{P},\ P(0) = P_0$

$$\boxed{P(t) = \frac{k^2t^2}{4} + kt\sqrt{P_0} + P_0}$$