MAT2705-04/05 20F Test 2 Print Name (Last, First) ___________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **You may use technology for row reductions, determinants and matrix inverses.** Otherwise only use technology to CHECK hand calculations, not subsitute for them, unless specifically requested. [Make sure you check every solution using Maple!]

pledge **[sign and date the pledge at the end of your exam]**

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

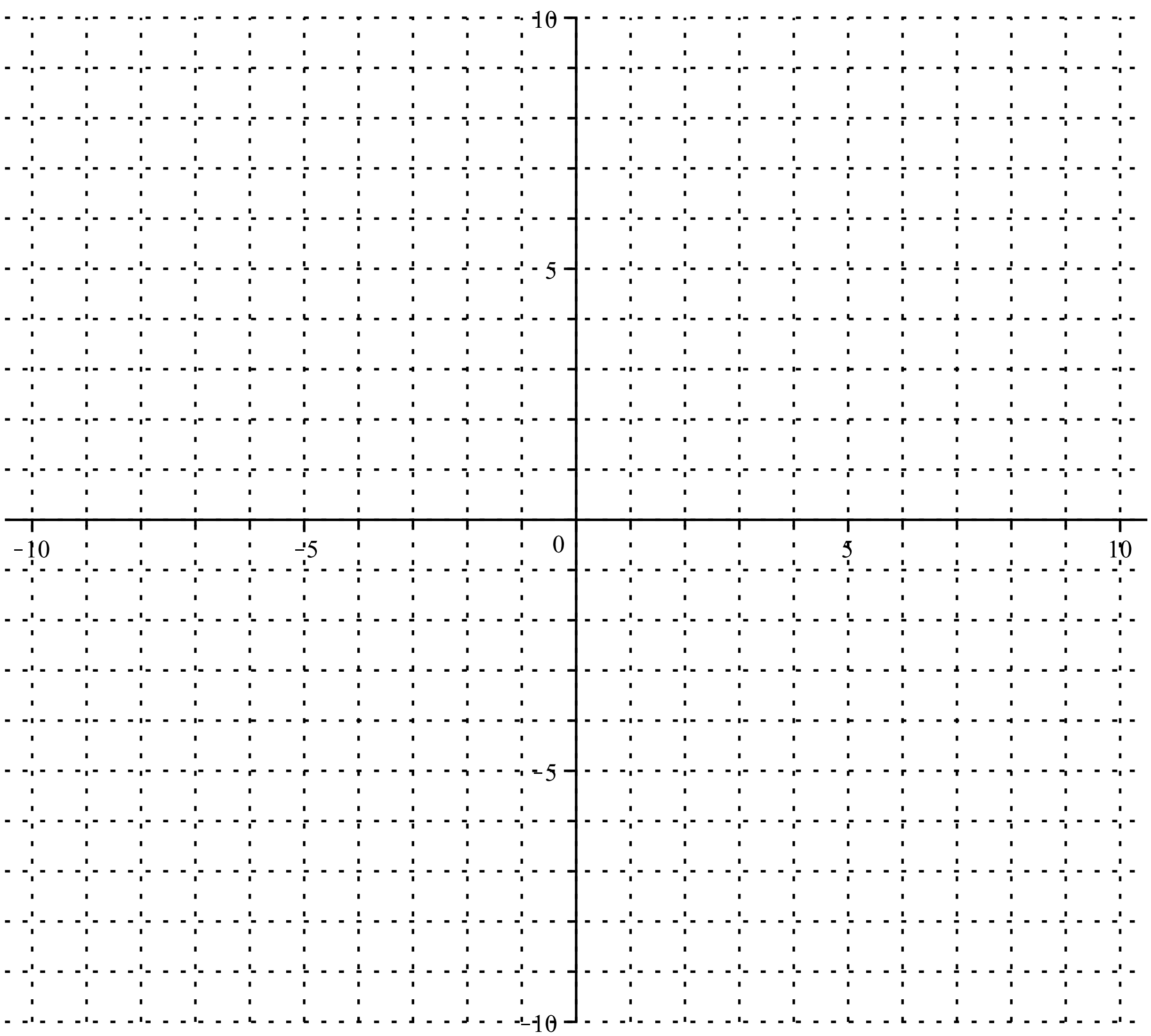
1. a) Express the following system in matrix form, evaluate the inverse coefficient matrix (technology!) and use it to solve the system
$4\,x_1+3\,x_2+2\,x_3=1,\ 5\,x_1+6\,x_2+3\,x_3=2,\ 3\,x_1+5\,x_2+2\,x_3=3.$
Box the scalar solutions (it consists of integers, in case you misenter the numbers and get ugly results).
b) Without having solved the system, what property does it have that guarantees it has a unique solution? Justify your claim.

2. Given $\begin{bmatrix} 1 & 1 & -1 & 7 \\ 1 & 4 & 5 & 16 \\ 1 & 3 & 3 & 13 \\ 2 & 5 & 4 & 23 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}=\begin{bmatrix} 1 \\ 10 \\ 7 \\ 11 \end{bmatrix}$ a) Find its general solution in vector form (integers!), documenting completely each step (identifying leading and free variables, etc), box it, then re-express the homogeneous part of the solution as an arbitrary linear combination of basis vectors of the solution space. What is the dimension of the related homogeneous solution space?
b) Identify the independent linear relationships among the columns $\langle \vec{v}_1 | \vec{v}_2 | \vec{v}_3 | \vec{v}_4 \rangle$ of the coefficient matrix which correspond to those basis vectors: $a\vec{v}_1+b\,\vec{v}_2+c\,\vec{v}_3+d\,\vec{v}_4=0$, etc. How many of these four vectors are linearly independent?
c) Express the right hand side of this matrix equation as a unique linear combination of the leading columns of the coefficient matrix.

3. a) On the grid below, **draw in** arrows representing the vectors $\vec{v}_1 = \langle 1, 2 \rangle$ and $\vec{v}_2 = \langle -2, 3 \rangle$ and $\vec{v}_3 = \langle 4, 1 \rangle$ and **label** them by their symbols. **Extend** the basis vectors $\{\vec{v}_1, \vec{v}_2\}$ to the corresponding coordinate axes for (y_1, y_2) and **mark** the positive direction with an arrow head and the axis label. Mark off tickmarks on these axes for integer values of the new coordinates.Then **draw in** the parallelogram with edges parallel to the new axes for which $\vec{v}_3$ is the main diagonal and shade it in in pencil lightly. Read off the coordinates (y_1, y_2) of $\vec{v}_3$ with respect to these two vectors (write them down) and **express** $\vec{v}_3$ as a linear combination of these vectors; **put this equation** at the tip of this vector.

b) Now use matrix methods to express $\vec{v}_3$ as a linear combination of the other two vectors (show all steps in this process), box it and then check your linear combination by expanding it out. Does your matrix result agree with your graphical result in part a)?

c) **Draw in** the arrow representing the vector $\vec{\boldsymbol{v}}_4$ whose new coordinates are $(y_1, y_2) = (-1, 1)$ and **label** the tip of $\vec{\boldsymbol{v}}_4$ by its symbol. Draw in the projection parallelogram associated with the new coordinates and lightly shade it in in pencil. Determine its old coordinates (x_1, x_2) graphically. Then evaluate them using a linear combination.



▶ **solution [put all other work and responses on separate sheets]**