MAT2705-04/05 20f Test 1 Print Name (Last, First) ___________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use **proper mathematical notation**, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not subsitute for them, unless specifically requested. [Make sure you check every solution using Maple!]

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page:

"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

1. Suppose the deer population $P(t)$ (with t in months) in a small forest initially numbers 25 and satisfies the initial value problem

$$\frac{dP}{dt} = 0.0225\,P - 0.0003\,P^2,\ P(0) = P_0\,.$$

a) What is the equilibrium population of deer?
b) Use Maple to state the general solution of this differential equation.
c) Looking at the exponential(s) in this solution, read off the exact value of the corresponding characteristic time and give its decimal approximation to 1 decimal place.
d) Solve this generic initial value problem using Maple (in terms of P_0).
e) Derive this same result by hand by solving the initial condition for your arbitrary constant, backsubsituting and simplifying.
f) Given your previous results, write down the solution with the concrete initial number of deer stated above and simplify the quotient coefficents by hand dividing out common multiples (this is an artificial problem with simple numbers!).
g) What percentage of the limiting deer population has been reached in 5 years?
h) How long does it take for the deer population to read 99 percent of the limiting population? [Does 1 percent of the limiting population really make sense in this context? Explain.]
i) Make a rough hand sketch of the concrete solution, including the lines representing the equilibrium solutions, labeling all key points (all the numbers from responding to previous parts) as well as locating the point $t = \tau$ on the horizontal axis, keeping the characteristic time in mind for the choice of window.

2. Consider the IVP: $\frac{dy}{dt} = y - t\,e^{-t},\ y(0) = \frac{1}{4}$.

a) Find the general solution of this differential equation by hand, step by step.
b) Find the IVP solution by hand, step by step.
c) Find the exact values of (t, y) where this solution has an obvious global maximum value for $t \geq 0$, and their decimal approximations to three decimal places.
d) Make a rough hand plot of the function for $t \geq 0$ showing all of its key features, keeping in mind the characteristic time considerations for this exponential.

3. Assume that a body moving with velocity v encounters resistance of the form $\frac{dv}{dt} = -k\, v^{\frac{3}{2}}$.

a) Show that $v(t) = \frac{4\, v_0}{\left(k\, t\, \sqrt{v_0} + 2\right)^2}$ by solving this DE by hand, step by step.

b) and that $x(t) = x_0 + \frac{2}{k} \sqrt{v_0} \cdot \left(1 - \frac{2}{k\, t\, \sqrt{v_0} + 2}\right)$, again showing steps by hand.

c) What is the limiting displacement (change in position) as $t \to \infty$?
[This shows that with a 3/2 power function resistance model, a body only coasts a finite distance before coming to a stop.]

d) If the initial velocity is 25 ft/minute and after a time interval of $\frac{1}{2}$ minute, the velocity has fallen to 16 ft/minute, what is the value of k? How far does the body coast?

e) How long does it take for the body to reach within 1 ft of the stopping location?

Maple hints:
Just use *x0* for x_0, no need for subscripts.
If you want to plot multiple expressions in x, you can use this plot command:

```
> plot([expression1, expression2,...], x=a..b, y=c..d, color=[red, blue,...], gridlines=true)
```

or you can copy and paste the additional expressions onto a PlotBuilder plot.