MAT2500-01/04 19S Quiz 7 Print Name (Last, First) ______________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

(See rewording on original!)   must be ≤ 9 ft   is the   for $2(x+y)+z$?

a) The girth of a rectangular package with smallest dimensions x and y ~~has a~~ cross-sectional perimeter of $2(x+y)$. ~~The girth is~~ the sum of this plus the longest dimension z. What are the dimensions and volume of the largest package that has the US Postal Service maximum of ~~a~~ 9 ft ~~girth~~? Identify the objective function and constraint and then describe the domain of the volume function $V(x,y)$ obtained by using the constraint to eliminate z (draw a diagram of this domain, identifying its edges) and confirm that your result is a local maximum of that function using the second derivative test. Locate your critical point in your diagram.
[HINT: The partial derivatives factor, making it easy to solve the relevant equations. Check with Maple.]
Finally answer the word problem with a complete English sentence describing completely your result, and give the result for the dimensions in inches as well as ft.

b) For the mathematical function $V(x,y)$, identify the remaining critical points and classify them as local max/min or saddle points.

dimensions so

a) $2(x+y)+z=9$ constraint $\rightarrow$ solve for z: $z = 9-2(x+y) > 0$, $x>0$, $y>0$

$V = xyz$ objective function $\longrightarrow$ $\boxed{V(x,y) = xy(9-2(x+y)) = 9xy-2x^2y-2xy^2}$

$x\neq 0 \neq y$ so only right factors can be 0.

$V_x = 9y-4xy-2y^2 = y(9-4x-2y) = 0$

$V_y = 9x-2x^2-4xy = x(9-2x-4y) = 0$

$4x+2y=9 \rightarrow 8x+4y=18$

$2x+4y=9 \quad\quad 2x+4y=9$

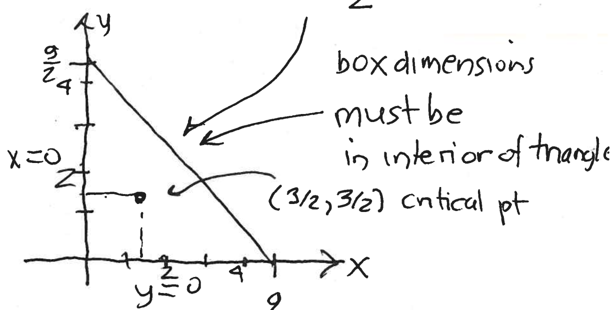
$6x=9 \rightarrow x=3/2$

$y = \frac{9-4x}{2} = \frac{9-4(3/2)}{2} = 3/2$

$z = 9-2(3/2+3/2) = 3$

$(x,y,z) = (\frac{3}{2},\frac{3}{2},3)$, $V = \frac{3}{2}\frac{3}{2}3 = \frac{27}{4} = 6\frac{3}{4}$

$2x+2y=9 \rightarrow y = \frac{9-2x}{2}$



$V_{xx} = -4y$

$V_{yy} = -4x$

$V_{xy} = 9-4x-4y$

$\boxed{\text{The largest box allowed has lateral dimensions 1.5 ft by 1.5 ft and length 3 ft with volume 6.75 ft}^3.}$

$\boxed{\text{namely 18 inches x 18 inches by 36 inches}}$

other crits: $y(9-4x-2y)=0 \rightarrow y=0$ or $9-4x-2y=0 \rightarrow 9-2y=0 \rightarrow y=9/2=4.5$

$x(9-2x-4y)=0 \rightarrow x=0$ or $9-2x-4y=0 \longrightarrow 9-2x=0 \quad x=9/2=4.5$

$x=0, y=0$

	$(3/2,3/2)$	$(0,0)$	$(0,9/2)$	$(9/2,0)$
V_{xx}	$-6<0$ ∩	0	$-18<0$ ∩	0
V_{yy}	$-6<0$ ∩	0	0	$-18<0$ ∩
V_{xy}	-3	9	-9	-9
$V_{xx}V_{yy}-V_{xy}^2$	$36-9>0$	$-81<0$	$-81<0$	$-81<0$
	confirms local max	saddle	saddle	saddle