MAT2500-01/04 19s Quiz 5 Take Home Print Name (Last, First) ______________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC).

1. $f(x,y,z) = x^2 y + x\sqrt{1+z}$

a) Evaluate the linear approximation at the point (1,2,3) without simplifying it by expanding out the expression.

b) Use it to approximate the function at the point $(1.1, 1.9, 3.2)$.

c) Compare to the exact value by evaluating the percentage error in using the linear approximation instead of the original function (new minus old over old).

d) Use the linear approximation to obtain the simplified standard equation for the tangent plane to the level surface $f(x,y,z) = f(1,2,3)$ by instead obtaining the level surface for the linear approximation function and then identify the components of the upward normal $\vec{n}$ which follow from that equation.

2. $z = \sin(x + \sin(t))$.

Show that $\dfrac{\partial z}{\partial x}\dfrac{\partial^2 z}{\partial x\,\partial t} = \dfrac{\partial z}{\partial t}\dfrac{\partial^2 z}{\partial x^2}$.

▶ solution

①a) $f(xyz) = x^2y + x(1+z)^{1/2}$

$f_x(xyz) = \frac{\partial}{\partial x}(x^2y + x(1+z)^{1/2}) = 2xy + (1+z)^{1/2}$

$f_y(xyz) = \frac{\partial}{\partial y}(x^2y + x(1+z)^{1/2}) = x^2$

$f_z(xyz) = \frac{\partial}{\partial z}(x^2y + x(1+z)^{1/2}) = x\frac{1}{2}(1+z)^{-1/2}(0+1) =$

$= \dfrac{x}{2(1+z)^{1/2}}$

$f_x(1,2,3) = 2(1)2 + (4)^{1/2} = 6$

$f_y(1,2,3) = 1$

$f_z(1,2,3) = \frac{1}{2\cdot 2} = \frac{1}{4}$ $\qquad f(1\,2\,3) = 1(2) + 1\cdot(2) = 4$

$\boxed{L(x,y,z) = 4 + 6(x-1) + 1(y-2) + \frac{1}{4}(z-3)}$

b) $L(1.1, 1.9, 3.2) = 4 + 6(1.1-1) + (1.9-2) + \frac{1}{4}(3.2-3)$

$= 4 + 6(0.1) - 0.1 + 0.05 = \boxed{4.55}$

c) $f(1.1, 1.9, 3.2) \approx 4.55333$

$\dfrac{L(1.1,1.9,3.2) - f(1.1,1.9,3.2)}{f(1.1,1.9,3.2)} \approx -0.00073$

$\boxed{\text{The linear approximation is about 0.073 percent less than the original function.}}$

d) $L(x,y,z) = 4 + 6x - 6 + y - 2 + \frac{1}{4}z - \frac{3}{4} = L(1,2,3) = f(1,2,3)$ (same values at original pt)

$= 4 - 6 - 2 - \frac{3}{4} + 6x + y + \frac{1}{4}z = -\frac{19}{4} + 6x + y + \frac{1}{4}z = 4$

$\boxed{6x + y + \frac{1}{4}z = \frac{35}{4}}$ or $\boxed{24x + 4y + z = 35}$

$\boxed{\vec{n} = \langle 24, 4, 1\rangle \text{ or } \langle 6, 1, 1/4\rangle}$

② $z = \sin(x + \sin t)$

$\frac{\partial z}{\partial x} = \cos(x+\sin t)\frac{\partial}{\partial x}(x + \sin t)$

$= \cos(x + \sin t)$

$\frac{\partial z}{\partial t} = \cos(x+\sin t)\underbrace{\frac{\partial}{\partial t}(x+\sin t)}_{=\cos t}$

$= \cos t \cdot \cos(x + \sin t)$

$\frac{\partial^2 z}{\partial t\,\partial x} = \frac{\partial}{\partial t}\cos(x+\sin t)$

$= -\sin(x+\sin t)\frac{\partial}{\partial t}(x+\sin t)$

$= -\cos t\,\sin(x+\sin t)$

$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\cos(x+\sin t)$

$= -\sin(x+\sin t)\frac{\partial}{\partial x}(x+\sin t)$

$= -\sin(x+\sin t)$

$\frac{\partial z}{\partial x}\frac{\partial^2 z}{\partial x\,\partial t} = -\cos(x+\sin t)\sin(x+\sin t)\cos t$

$\frac{\partial z}{\partial t}\frac{\partial^2 z}{\partial x^2} = \cos t\,\cos(x+\sin t)(-)\sin(x+\sin t)$

$= -\cos t\,\sin(x+\sin t)\cos(x+\sin t)$ ✓