MAT2500-01/04 19S Final Exam Print Name (Last, First) _____|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL signs and arrows when appropriate. Always SIMPLIFY expressions. LABEL parts of problem. BOX final short answers. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Use technology to evaluate any integrals you set up. JUSTIFY any numbers or equations that play a role in your calculations.

Recall: $\operatorname{curl}\left(\vec{F}\right) = \left\langle \frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right\rangle$

1. Green's Theorem: for a counterclockwise oriented curve C which is the boundary of R:

$$\int_C \vec{F} \cdot d\vec{r} = \iint_R \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \, dA$$

a) Use Green's Theorem to evaluate $\int_{C_1} x^2 y \, dx - x y^2 \, dy$, where C_1 is the circle $x^2 + y^2 = 4$ with counterclockwise orientation. Annotate your diagram of the curve and its interior to describe your double integration iteration. Integrate by hand.

b) Repeat for the circle C_2: $x^2 + y^2 = 4x$ with the counterclockwise orientation.

c) Using standard polar coordinates to parametrize the curve C_2, evaluate the line integral around it directly, evaluating the final integral in Maple.

2. $\vec{F}(x, y, z) = \langle x^2 y, y z, z x^2 \rangle$, $C : x = \cos(t), y = \sin(t), z = \sin(t), t = 0..2\pi$

Set up the line integral of this vector field along this parametrized curve, and then use Maple to evaluate it exactly after simplifying the integrand.

3. $\vec{F} = \langle 3x^2 y z - 3y, x^3 z - 3x, x^3 y + 2z \rangle$.

a) Evaluate the divergence of $\vec{F}$.

b) Show that $\operatorname{curl}\left(\vec{F}\right) = \vec{0}$, i.e., it is a conservative vector field.

c) Find a potential function $f(x, y, z)$ for $\vec{F}$.

d) Use the potential to evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ from the point $(1, 2, 3)$ to the point $(2, 3, 4)$.

e) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the straight line from between the same initial and final points. Use Maple to evaluate the integral.

Do your results for d) and e) agree as they should? Do you know what year the Wildcats won their first NCAA championship?

▶ solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in with your answer sheets as a cover page, with the Lastname, FirstName side face up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: