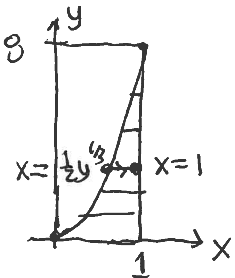
MAT2500-01/04 19S TakeHomeTest 3 Answers

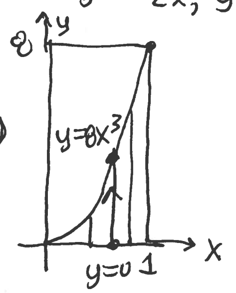
① a) $\displaystyle\int_{y=0}^{y=8}\int_{x=\frac{1}{2}y^{1/3}}^{x=1} \cos x^4\,dx\,dy$

$y^{1/3}=2x,\quad y=8x^3$



$y=0..8$

b)



$x=0..1$

c) $$\boxed{\int_0^1\int_0^{8x^3}\cos x^4\,dy\,dx}$$

$$= \int_0^1 y\cos x^4\Big|_{y=0}^{y=8x^3}dx$$

$$= \int_0^1 8x^3\cos x^4\,dx$$

$u=x^4$
$du=4x^3dx$

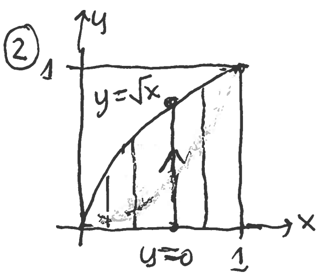
numerical value agrees

$$= \int_{x=0}^{x=1} \underbrace{2\cos u\,du}_{2\sin u}$$

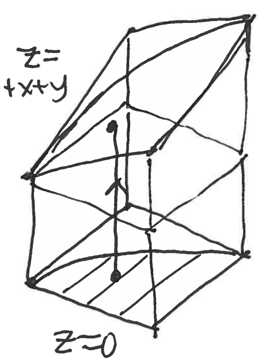
$$= 2\sin x^4\Big|_0^1 = \boxed{2\sin 1 \approx 1.6829}$$

d)

②



$y=0..\sqrt{x}$
while $x=0..1$



$z=0..1+x+y$

$$\iiint_E 6xy\,dV = \int_0^1\int_0^{\sqrt{x}}\int_0^{1+x+y} 6xy\,dz\,dy\,dx$$

$$= \quad 6xyz\Big|_{z=0}^{z=1+x+y} = 6xy(1+x+y) = 6(xy+x^2y+xy^2)$$

$$= \quad 6\left(\frac{xy^2}{2}+\frac{x^2y^2}{2}+\frac{xy^3}{3}\right)\Big|_{y=0}^{y=\sqrt{x}}$$

$$6\left(\frac{x\cdot x}{2}+\frac{x^2 x}{2}+\frac{x\,x^{3/2}}{3}\right) = 6\left(\frac{x^2}{2}+\frac{x^3}{2}+\frac{x^{5/2}}{3}\right)$$

② (continued)

$$\iiint_E 6xy\,dV = \int_0^1 6\left(\frac{x^2}{2}+\frac{x^3}{2}+\frac{x^{5/2}}{3}\right)dx$$

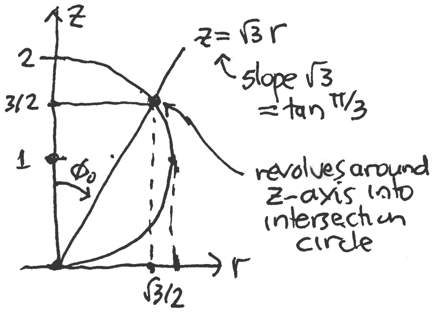
$$= 6\left(\frac{x^3}{6}+\frac{x^4}{8}+\frac{x^{7/2}}{\frac{7}{2}\cdot 3}\right)\Big|_0^1 = 6\left(\frac{1}{6}+\frac{1}{8}+\frac{2}{21}\right) = \boxed{\frac{65}{28}}$$

≈ 2.321

③ a) $z^2 = 3(x^2+y^2) = 3r^2 \rightarrow z=\sqrt{3}\,r.\quad (z\ge 0)$

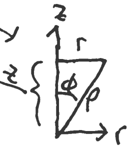
$x^2+y^2+(z-1)^2=1 \rightarrow r^2+(z-1)^2=1$

circle at $(r,z)=(0,1)$ center
radius 1



$$\underbrace{r^2+z^2}_{\rho^2}-2\underbrace{z}_{\rho\cos\phi}+\cancel{1}=\cancel{1}$$

$\hookrightarrow \rho = 2\cos\phi$



intersection: $z=\sqrt{3}\,r$
$r^2+z^2=2z$
$r^2+3r^2=2\sqrt{3}\,r$
$4r^2=2\sqrt{3}\,r \quad r=\frac{\sqrt{3}}{2},0$
$z=\frac{3}{2},0$

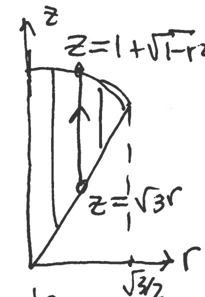
$\tan\phi_0 = \frac{r}{z} = \frac{r}{\sqrt{3}r} = \frac{1}{\sqrt{3}}$

$\phi_0 = \arctan\frac{1}{\sqrt{3}} = \pi/6$

$\rho = 2\cos\phi_0 = 2\cos\pi/6 = 2\sqrt{3}/2 = \sqrt{3}$

intersection circle: $\boxed{\begin{array}{l}(r,z)=\left(\frac{\sqrt{3}}{2},\frac{3}{2}\right)\\ (\rho,\phi)=(\sqrt{3},\pi/6)\end{array}}$

b)



$z^2-2z+r^2=0$

$z=\underbrace{\frac{2\pm\sqrt{4-4r^2}}{2}}=1\pm\sqrt{1-r^2}$

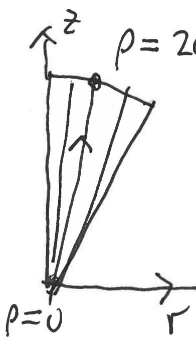
uppersphere $z=1+\sqrt{1-r^2}$

$r=0..\sqrt{3}/2 \qquad \theta=0..2\pi$ rotational symmetry.

$$\langle V,V_z\rangle = \int_0^{2\pi}\int_0^{\sqrt{3}/2}\int_{\sqrt{3}r}^{1+\sqrt{1-r^2}} \langle 1,z\rangle\, r\,dz\,dr\,d\theta \overset{\text{Maple}}{=} \left\langle \frac{7\pi}{12},\frac{37\pi}{48}\right\rangle$$

$$\bar{z} = \frac{V_z}{V} = \frac{37\pi/48}{7\pi/12} = \boxed{\frac{37}{28} \approx 1.321}$$

c)

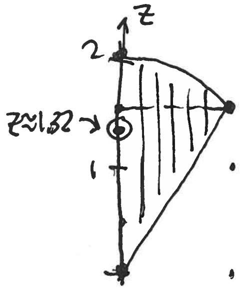


$\rho=0..$
$\phi=0..\pi/6$
$\theta=0..2\pi$

3 c) continued

$$\langle V, Vz\rangle = \int_0^{2\pi}\int_0^{\pi/6}\int_0^{2\cos\phi} \langle 1, \overbrace{\rho\cos\phi}^{z}\rangle \underbrace{\rho^2\sin\phi}_{\text{correction factor for } dV}\, d\rho\, d\phi\, d\theta = \text{Maple } \left\langle \frac{2\pi}{12}, \frac{37\pi}{48}\right\rangle$$

same as before. so same centroid values ✓



should be below spherical cap but above midheight. looks reasonable.

b) see test sheet and below.

4 a)

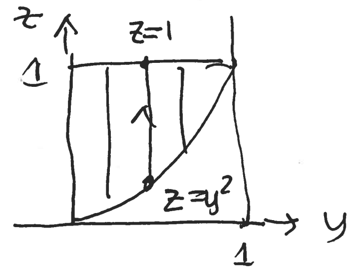
$$\underbrace{\int_{y=0}^{y=1}\int_{z=y^2}^{z=1}}_{\text{outer double int}}\underbrace{\int_{x=0}^{x=y^{1/3}} 1\, dx}_{\text{inner integral}}\, dz\, dy =$$

$$x\Big|_{x=0}^{x=y^{1/3}} = y^{1/3}$$

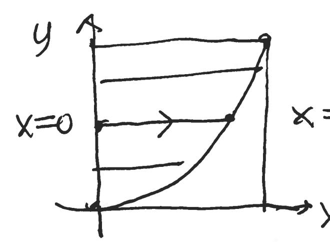
$$y^{1/3}z\Big|_{z=y^2}^{z=1} = y^{1/3}(1-y^2) = y^{1/3} - y^{7/3}$$

$$\frac{3}{4}y^{4/3} - \frac{3}{10}y^{10/3}\Big|_{y=0}^{y=1} = \frac{3}{4} - \frac{3}{10} = \frac{3}{2}\left(\frac{1}{2}-\frac{1}{5}\right) = \boxed{\frac{9}{20}}$$

c)

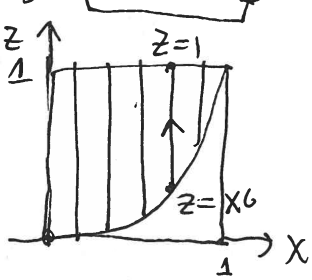


$y = 0..1$



$x = y^{1/3} \to y = x^3$

d) $y = x^3 \cap z = y^2 = (x^3)^2 = x^6$



$x = 0..1$

outer double integral

$$\boxed{\int_0^1\int_{x^6}^1\int_{x^3}^{z^{1/2}} 1\, dy\, dz\, dx = \frac{9}{20}}$$

innermost integral y first:

$y = x^3 .. z^{1/2}$

side wall .. bottom side

see diagram on test sheet.

b) see test diagram.
The equations of the surfaces which bound the solid region are:

ceiling:	$z=1$
floor/sidewall:	$z=y^2$
back wall:	$x=0$
leftside wall:	$x=y^{1/3}$ or $y=x^3$

Most students listed boundaries of each face but not the equation of the face surface! I did not grade this.

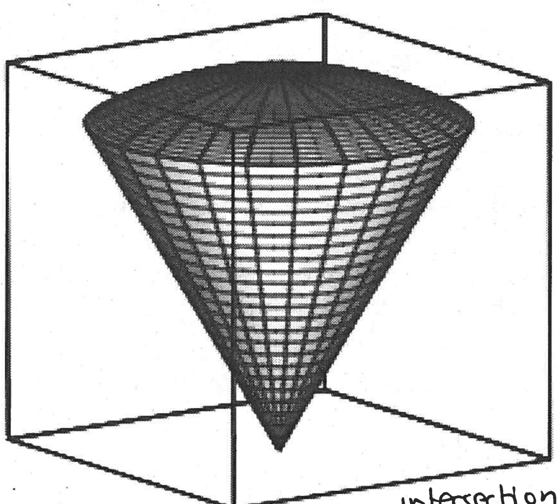
c) Make a labeled plane diagram describing the outer double integral (with a labeled cross-section for the inner integration of the outer double integral) and draw into the existing diagram a labeled cross-section for the innermost integral), in each case labeling the cross-section by the starting and stopping value equations for the variable of integration (appropriate arrowhead midway!).

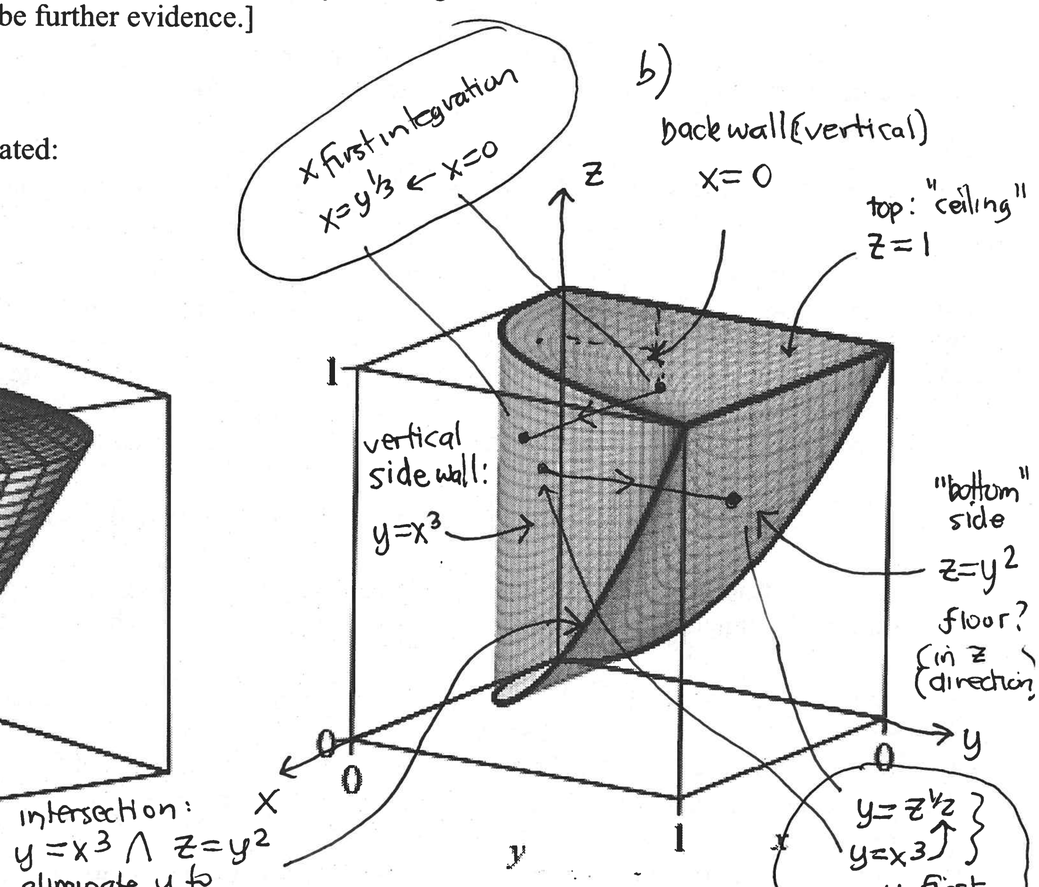
d) Rewrite the integral in the order $\iiint \ldots \, dy\, dz\, dx$, supporting your limits of integration with a labeled cross-section in the existing diagram and a plane diagram describing the new outer double integral.

e) Use Maple to evaluate the latter integral and compare to your original value. Your results should agree. [This is not a guarantee that your two interations are correct. If you also get the same result for both iterations for integrating z instead of 1, this would be further evidence.]

▶ solution

Problems 3, 4 integral regions illustrated:





the origin is in the back bottom of cube

No collaboration. You may only talk to bob. See test rules on-line. Read short rules above. Print out and attach any Maple supporting work you do, hand annotating if necessary with problem number and part etc, whatever is necessary for clarification.

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in stapled to your answer sheets as the cover page, with the Lastname, FirstName side face up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: