MAT2500-01/04 19S Take Home Test 3   Print Name (Last, First) ___________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Explain in as many words as possible everything you are doing! For each hand integration step, state the antiderivative formula used before substituting limits into it:

$\int_a^b f(x)\ \mathrm{d}x = F(x)\big|_{x=a}^{x=b} = F(b) - F(a)$. Every integral should be checked with Maple. Do not read off answers from technology unless explicitly requested, otherwise use it only to CHECK your hand calculations. All interated integral calculations must be supported by "cross-section shaded" labeled cross-section diagrams.

1. a) Deconstruct the integral $\int_0^8 \int_{\frac{1}{2} y^{\frac{1}{3}}}^{1} \cos(x^4)\ \mathrm{d}x\ \mathrm{d}y$, namely create a diagram of the region of integration with a bullet point delimited linear cross-section (appropriate arrow head in the middle) annotated to represent the inner partial integration.

b) Redo the diagram for the opposite order partial integration in the iteration.

c) Write down and evaluate by hand exactly the resulting integral.

d) Compare the numerical values of the two definite integrals to make sure they agree.

2. Evaluate step by step the integral $\iiint_E 6\,x\,y\ \mathrm{d}V$, where E lies under the plane $z = 1 + x + y$ and above the region in the x y-plane bounded by the curves $y = \sqrt{x}$, $y = 0$ and $x = 1$. Support your work by a rough labeled 3d diagram representing the innermost integral, and a 2d diagram representing the outer double integral.

3. Consider the solid region $z^2 = 3(x^2 + y^2)$, $x^2 + y^2 + (z-1)^2 = 1$, $z \geq 0$ inside the sphere and the upper cone, forming an ice cream cone shaped solid.

a) Describe the boundary of this region in cylindrical and spherical coordinates, and make a diagram in the r-z half-plane that illustrates this solid of revolution. Find the values of (r, z, ρ, ϕ) for the intersection circle which is a point in this plane.

b) Set up the two iterated integrals in cylindrical coordinates needed to find the centroid, or geometric center of the region,which obviously lies on the z-axis by symmetry, justifying your limits of integration with an annotated cross-sectional r-z halfplane diagram, and evaluate those integrals exactly with Maple. Then evaluate the centroid exactly and numerically to 2 decimal places.

c) Repeat for spherical coordinates, including a new annotated cross-sectional r-z halfplane diagram which describes the iteration of the integral.

d) Locate your centroid point on the r-z half-plane diagram. Does it look reasonable for the solid of revolution? Any thoughts about why or why not?

4. Consider the solid region R in the first octant corresponding to the region of integration in the triple integral

$\int_0^1 \int_{y^2}^1 \int_0^{y^{\frac{1}{3}}} 1\ \mathrm{d}x\ \mathrm{d}z\ \mathrm{d}y$. See the figure on page 2.

a) Evaluate this integral step by step (and check with Maple!).

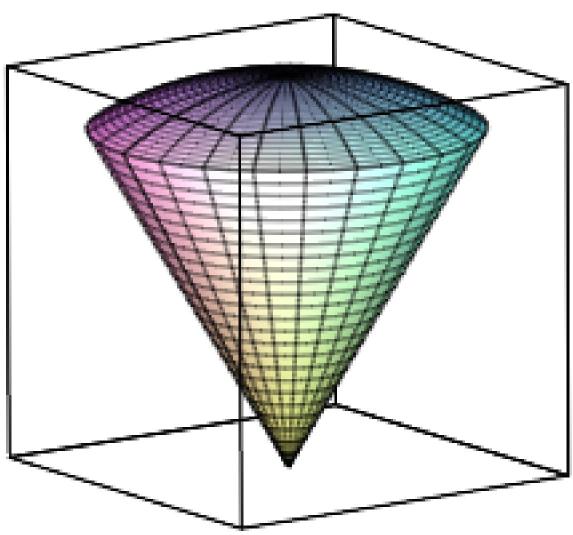
b) Describe the 4 faces of the solid region of integration by their equations: the vertical back and front walls (looking down towards the origin in the first octant as in the figure) and the ceiling and "floor".

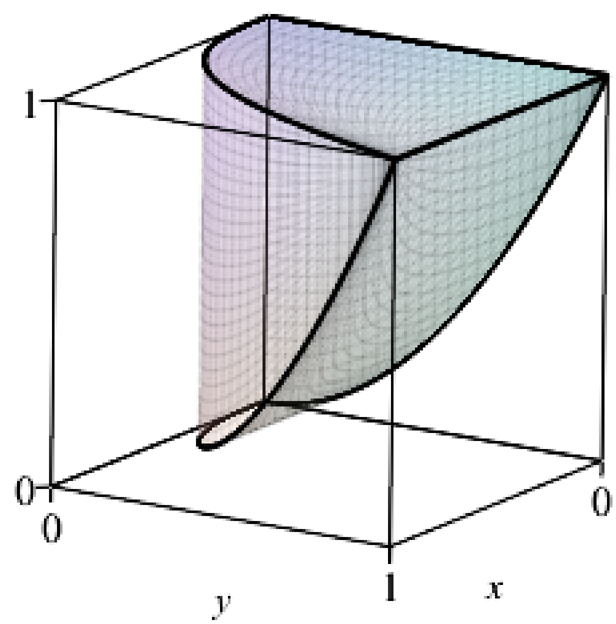
c) Make a labeled plane diagram describing the outer double integral (with a labeled cross-section for the inner integration of the outer double integral) and draw into the existing diagram a labeled cross-section for the innermost integral), in each case labeling the cross-section by the starting and stopping value equations for the variable of integration (appropriate arrowhead midway!).

d) Rewrite the integral in the order $\iiint \ldots \, dy\, dz\, dx$, supporting your limits of integration with a labeled cross-section in the existing diagram and a plane diagram describing the new outer double integral.
e) Use Maple to evaluate the latter integral and compare to your original value. Your results should agree.
[This is not a guarantee that your two interations are correct. If you also get the same result for both iterations for integrating z instead of 1, this would be further evidence.]

▶ solution

Problems 3, 4 integral regions illustrated:





the origin is in the back bottom of cube

No collaboration. You may only talk to bob. See test rules on-line. Read short rules above. Print out and attach any Maple supporting work you do, hand annotating if necessary with problem number and part etc, whatever is necessary for clarification.

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in stapled to your answer sheets as the cover page, with the Lastname, FirstName side face up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: