Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type(Maple, GC). You are encouraged to use technology to CHECK but NOT SUBSTITUTE all of your hand results. **Everything on this test is straightforward to evaluate by hand and must be shown explicitly**.

1. Identify each quantity you evaluate with its proper symbol and defining formula.
a) Find the direction $\hat{\boldsymbol{u}}$ in which $f(x, y, z) = z\,e^{xy}$ increases most rapidly at the point (0,1,2).
b) What is the maximum rate of increase there?
c) What is the directional derivative in the direction of the origin from this point?
d) Use the chain rule to evaluate the derivative of f along the almost twisted cubic $(x, y, z) = \left(2\,t - 2, t^2, 2\,t^3 \right)$ as it passes through the point (0,1,2).
e) Write a simplified (standard linear form) equation for the tangent plane to the level surface at this point. What is the equation of the level surface itself through this point? Evaluate the linear approximation $L(x, y, z)$ at this point.

2. A mathematical "sports dome" for Villanova has the shape of an ellipsoidally deformed hemisphere (upper half of an ellipsoid with axes aligned with a Cartesian coordinate system). The volume of this half ellipsoid is $V = \frac{2}{3}\pi\, a\, b\, c$ in terms of the 3 semiaxes with the same (semiaxis) dimensions as Villanova's planned dome with a more rectangularly shaped base: 38'x110'x185' (in feet, according to *The Villanovan*). So let:

$(a, b, c) = (185, 110, 38) = (\mathit{halflength}, \mathit{halfwidth}, \mathit{height})$.

Suppose a waterproofing layer is 1/8" (in inches) thick when applied to the dome. Use differentials to estimate the change in volume in cubic feet (to the nearest cubic foot) if we increase the dimensions by this amount, which is an estimate of the volume of waterproofing material needed.

3. The roof of this mathematical sports dome has some internal spotlights (for night use) mounted so that they are pointing down perpendicular to the surface.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, z \geq 0 .$$

The point on the dome above the floor $z = 0$ at $\left(x_0, y_0\right) = (100, 50)$ is approximately at $z_0 = 26.9$ to high accuracy. Write down an equation for the normal line to this point on the dome and evaluate the location $\left(x_s, y_s\right)$ where it hits the floor, i.e., where the center of this spotlight hits the floor. Give your results to the nearest tenth of a foot.
[Hint: work the problem in terms of the parameters and insert the numbers at the end to simplify matters.]
Optional Comment. Does your result make sense to you in comparison with the horizontal location of the spotlight, given the concave downward curvature of the dome? If so, why?

▶ solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: