MAT2500-01/04 19S Test 1 Answers

a) $\vec r = \langle t^2, 9t, 4t^{3/2}\rangle$ $\quad (t \ge 0 \text{ for real } t^{3/2})$

$\vec v = \vec r\,' = \langle 2t, 9, 6t^{1/2}\rangle \qquad |\vec r\,'| = \sqrt{4t^2+81+36t} = \sqrt{(9+2t)^2} = |9+2t| = 9+2t$ for $t \ge 0$ $= v$

$\vec a = \vec r\,'' = \langle 2, 0, 3t^{-1/2}\rangle \qquad |\vec r\,''| = \sqrt{4+9t^{-1}} = a$

$\hat T = \dfrac{\vec r\,'}{|\vec r\,'|} = \dfrac{\langle 2t, 9, 6t^{1/2}\rangle}{9+2t}$

b) $\vec r = \underbrace{\vec r_0}_{\vec r(1)} + t\underbrace{\vec a}_{r'(1)} = \langle 1,9,4\rangle + t\langle 2,9,6\rangle = \boxed{\begin{aligned}&\langle 1+2t, 9+9t, 4+6t\rangle\\ &= \langle x,y,z\rangle\end{aligned}}$

$\vec r(1) = \langle 1,9,4\rangle$

$\vec r\,'(1) = \langle 2,9,6\rangle$

$\vec r\,''(1) = \langle 2,0,3\rangle$

$\hat T(1) = \dfrac{\langle 2,9,6\rangle}{11}$

$v(1) = 11$

$a(1) = \sqrt{13}$

c) $\vec r\,' \times \vec r\,'' = \langle 2t, 9, 6t^{1/2}\rangle \times \langle 2, 0, 3t^{-1/2}\rangle$

$\vec b \parallel \underset{\text{Maple}}{=} \boxed{\langle 27t^{-1/2}, 6t^{1/2}, -18\rangle} = 3\langle 9t^{-1/2}, 2t^{1/2}, -6\rangle$

$\boxed{\vec b(1) = \langle 27, 6, -18\rangle} = 3\underbrace{\langle 9, 2, -6\rangle}_{\vec n}$

$|\vec b(1)| = 3\sqrt{81+4+36} = 3\sqrt{121} = 3(11) = \boxed{33}$

d) $\vec n \cdot (\vec r - \underbrace{\vec r_0}_{\vec r(1)}) = 0 \;\to\; \langle 9,2,-6\rangle \cdot \langle x-1, y-9, z-4\rangle = 0$

$9(x-1) + 2(y-9) - 6(z-4) = 0$

$9x + 2y - 6z \underbrace{-9-18+24}_{-3} = 0$

$\boxed{9x + 2y - 6z = 3}$

e) $K = \dfrac{|\vec r\,' \times \vec r\,''|}{|\vec r\,'|^3} = \boxed{\dfrac{3\sqrt{81t^{-1}+4t+36}}{(9+2t)^3}}$ $\qquad K(1) = \dfrac{3\sqrt{81+4+36}}{11^3} = \dfrac{3\sqrt{121}}{11^3} = \dfrac{3(11^2)^{1/2}}{11^3} = \dfrac{3}{11^2} = \boxed{\dfrac{3}{121}}$

perfect square!

but perfect square again!

$\rho = 1/K = \boxed{\dfrac{(9+2t)^3}{3\sqrt{81t^{-1}+4t+36}}}$ $\qquad$ so $= \dfrac{3\sqrt{\dfrac{4t^2+81+36t}{t}}}{(9+2t)^3} = \dfrac{3(2t+9)}{(2t+9)^3\,t^{1/2}} = \boxed{\dfrac{3}{(2t+9)^2\,t^{1/2}}}$

$= \boxed{\dfrac{(2t+9)^2\,t^{1/2}}{3}} \qquad \boxed{\rho(1) = \dfrac{121}{3}}$

note $|\vec r\,' \times \vec r\,''| = \dfrac{3}{t^{1/2}}\sqrt{4t^2+81+36t} = \dfrac{3}{t^{1/2}}(9+2t) = |\vec b(t)|$

f) $\hat B = \dfrac{\vec b}{|\vec b|} = \dfrac{\langle 27t^{-1/2}, 6t^{1/2}, -18\rangle\, t^{1/2}}{3(9+2t)} = \dfrac{\langle 27, 6t, -18t^{1/2}\rangle}{3(9+2t)} = \boxed{\dfrac{\langle 9, 2t, -6t^{1/2}\rangle}{(9+2t)}}$ $\quad \boxed{\hat B(1) = \dfrac{\langle 9, 2, -6\rangle}{11}}$

g) $\hat N = \hat B \times \hat T = \dfrac{\langle 9, 2t, -6t^{1/2}\rangle}{(9+2t)} \times \dfrac{\langle 2t, 9, 6t^{1/2}\rangle}{(9+2t)} \underset{\text{Maple}}{=} \dfrac{\langle \underbrace{12t^{3/2}+54t^{1/2}}_{6(9+2t)t^{1/2}}, \underbrace{-54t^{1/2}-12t^{3/2}}_{-6(9+2t)t^{1/2}}, \underbrace{-4t^2+81}_{(9-2t)(9+2t)}\rangle}{(9+2t)^2}$

perfect square factors out!!

$= \boxed{\dfrac{\langle 6t^{1/2}, -6t^{1/2}, 9-2t\rangle}{9+2t}} \quad \boxed{\hat N(1) = \dfrac{\langle 6, -6, 7\rangle}{11}}$

h) $a_T(1) = \hat T(1) \cdot \vec a(1) = \dfrac{\langle 2,9,6\rangle}{11} \cdot \langle 2,0,3\rangle = \dfrac{4+18}{11} = \dfrac{22}{11} = \boxed{2}$

$a_N(1) = \hat N(1) \cdot \vec a(1) = \dfrac{\langle 6,-6,7\rangle}{11} \cdot \langle 2,0,3\rangle = \dfrac{12+21}{11} = \dfrac{33}{11} = \boxed{3}$

i) $L = \int_0^2 |\vec r\,'(t)|\,dt = \int_0^2 9+2t\,dt = 9t+t^2\Big|_0^2 = 18+4 = \boxed{22}$

j) $\vec C(1) = \vec r(1) + \rho(1)\hat N(1) = \langle 1,9,4\rangle + \underbrace{\dfrac{121}{3}\dfrac{\langle 6,-6,7\rangle}{11}}_{\langle 22, -22, \frac{77}{3}\rangle} = \langle 23, -13, \underbrace{4+\tfrac{77}{3}}_{\frac{89}{3}}\rangle$

NOTE: "parametrized" eqns for a line is an equation(s): They are not just the RHS.

$\underset{\vec r = \langle x,y,z\rangle}{\text{LHS}} \overset{\text{equation!}}{=} \underbrace{\text{RHS}}_{\text{vector function of parameter}}$

obvious factors of 3 should be cancelled to simplify expressions.