Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). You are encouraged to use technology to check all of your hand results.
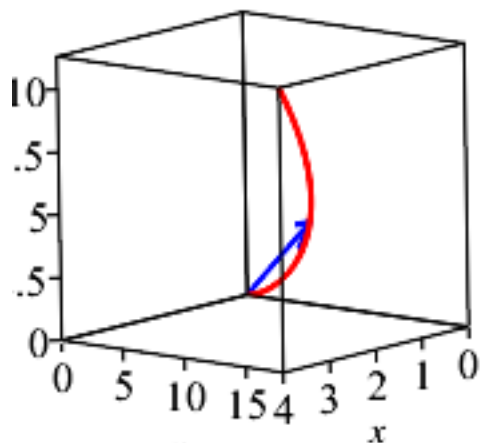
The parametrized curve segment $\vec{r}(t) = \left\langle t^2, 9t, 4t^{\frac{3}{2}} \right\rangle, 0 \le t \le 2$
is shown in the figure together with $\vec{r}(1)$.
[This is a perfect square speed case! factor!]
a) Evaluate and simplify $\vec{v}(t) = \vec{r}\,'(t), \vec{a}(t) = \vec{r}\,''(t), v(t) = |\vec{r}\,'(t)|$,
$\vec{T}(t), a(t) = |\vec{r}\,''(t)|$ and their values (including $\vec{r}(t)$) at $t = 1$.
b) Write the parametrized equations of the tangent line through $\vec{r}(1)$.
c) Evaluate and simplify $\vec{b}(t) = \vec{r}\,'(t) \times \vec{r}\,''(t)$. Evaluate $\vec{b}(1)$ and
$|\vec{b}(1)|$.

d) Write the equation of the osculating plane through $\vec{r}(1)$ containing the tangent vector and the second derivative there.

e) Evaluate the curvature $\kappa(t) = \dfrac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$ and its reciprocal, the radius of curvature $\rho(t)$ and $\rho(1)$.

f) Evaluate and simplify the unit vector $\vec{B}(t)$ in the direction of $\vec{b}(t) = \vec{r}\,'(t) \times \vec{r}\,''(t)$ and then $\vec{B}(1)$.

g) Evaluate and simplify the unit normal $\vec{N}(t) = \vec{B}(t) \times \vec{T}(t)$.

h) Evaluate the scalar tangential projection $a_T(1)$ along $\vec{T}(1)$ of the acceleration $\vec{a}(1) = \vec{r}\,''(1)$ and its scalar normal projection $a_N(1) = \vec{N}(1) \cdot \vec{a}(1)$ exactly. [They are integers!]

i) Write down an integral formula for the length of the curve $\vec{r}(t)$ for $t = 0 .. 2$. Evaluate it exactly and then numerically to 4 decimal places.

j) **Optional.** The center of the osculating circle has position vector: $\vec{C}(t) = \vec{r}(t) + \rho(t) \vec{N}(t)$. Show that $\vec{C}(1) = \langle 23, -13, ... \rangle$.

▼ **pledge**

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

> $with(Student[VectorCalculus]):$ *# needed for dot product to work properly, and magnitude* $\|\vec{v}\|$

> $r(t) := \left\langle t^2, 9\,t, 4\,t^{\frac{3}{2}} \right\rangle :$

> $r(t); r'(t); r''(t)$

► **solution**