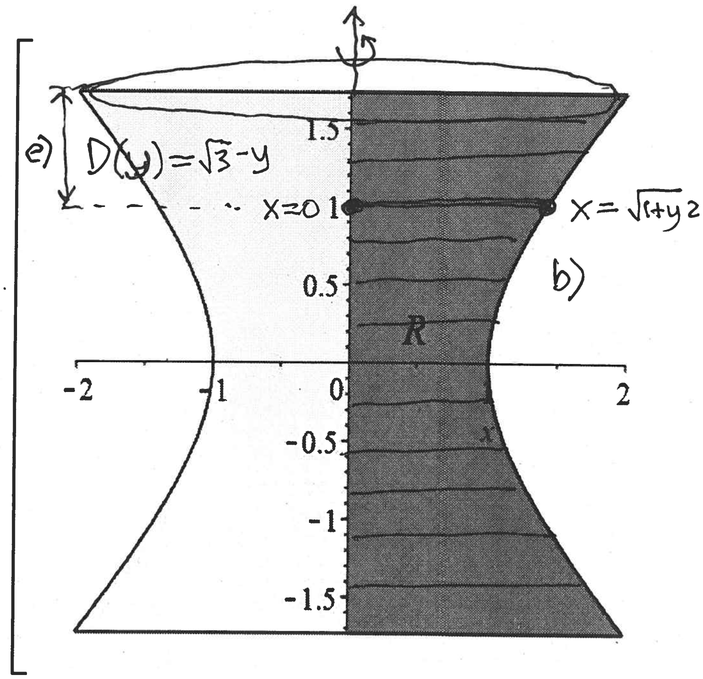
MAT1505-03/04 19F Quiz 4 Print Name (Last, First) ________________, ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

A cooling tower (think Limerick) has the following shape: revolve around the y axis the region R between the right half of the hyperbola $x^2 - y^2 = a^2$ cut off by the vertical line $x = a + h$ and the y axis, first choosing $a = 1, h = 1$ to have concrete numbers to work the problem.

a) Where does the vertical line intersect the hyperbola, i.e. , what are the (x, y) coordinates of those two obvious points in the right half of the plane as shown in the diagram.

b) Shade in only R with equally spaced cross-sections perpendicular to the axis of symmetry, and identify the radius $r(y)$ on a typical such cross-section, labeling its bullet endpoints with the starting and stopping value equations.



c) Rotate this region around the axis y axis to form an hourglass solid region on its side. Write down an integral V_1 (with simplified integrand) representing the volume of the resulting solid of revolution.

d) Evaluate this integral exactly using Maple and then approximate the result to 4 decimal places.

e) Write down an integral representing the work needed to move each layer of this solid at position y from its present position to the top by a constant horizontal force of 1 force (weight!) unit per unit volume (like unit gravity, setting $\rho = 1$).

i.e., $W = \rho \int_a^b A(y)\, D(y)\, dy$, where $A(y)$ is the cross-sectional area and $D(y)$ is the displacement each layer at position y undergoes).

f) Evaluate this integral exactly using Maple and then approximate the result to 4 decimal places.

a) $x^2 - y^2 = 1 \xrightarrow{x=2} 4 - y^2 = 1,\ y^2 = 3,\ y = \pm 3$ pts $\boxed{(2, \pm\sqrt{3}) = (x,y)}$ $\sqrt{3} \approx 1.73$ ✓

b) $x^2 = 1 + y^2 \rightarrow x = \pm\sqrt{1+y^2} \rightarrow (x>0): x = \sqrt{1+y^2}$

c) $R(y) = \sqrt{1+y^2}$, $A(y) = \pi R(y)^2 = \pi(1+y^2)$ $V_1 = \int_{-\sqrt{3}}^{\sqrt{3}} A(y)\,dy = \boxed{\int_{-\sqrt{3}}^{\sqrt{3}} \pi(1+y^2)\,dy}$

e) $\boxed{W = 1 \int_{-\sqrt{3}}^{\sqrt{3}} (\sqrt{3}-y)(\pi)(1+y^2)\,dy}$

d) $\underset{\text{Maple}}{=}$ $\boxed{4\pi\sqrt{3} \approx 21.7656}$

f) $\boxed{= 12\pi \approx 37.6991}$

g) $x^2 - y^2 = a^2 \rightarrow y^2 = (a+h)^2 - a^2 = 2ah + h^2$

$x = a + h$ $\quad y = \pm\sqrt{2ah+h^2}$

$x^2 = a^2 + y^2 \rightarrow x = \pm\sqrt{a^2+y^2} \xrightarrow{x>0} x = \sqrt{a^2+y^2}$

$A(y) = \pi(a^2+y^2)$

$V = \int_{-\sqrt{2ah+h^2}}^{\sqrt{2ah+h^2}} \pi(a^2+y^2)\,dy$

$W = \int_{-\sqrt{2ah+h^2}}^{\sqrt{2ah+h^2}} (\sqrt{2ah+h^2} - y)(\pi)(a^2+y^2)\,dy$

(over)

g) **Optional.** Now repeat this calculation for the original parameters $a, h > 0$, and factor the result (Maple) to simplify it.
h) **Optional reflection (translation: ignore this).** If you concentrate all the weight at the obvious center of the 3d solid region of revolution, the midpoint of the rotation axis, we could move it to the "top" a distance of half the axis length. Compare that work done this way (volume times half axis displacement) with the result of part e). Interesting, no? [You will learn about center of mass in calc 3.]

▶ solution

g) continued Maple: $V = \frac{2\pi}{3}\sqrt{2ah+h^2}\,(3a^2+2ah+h^2)$

$W = \frac{2\pi}{3}(2ah+h^2)(3a^2+2ah+h^2)$

h) equivalent to: $= \underbrace{\sqrt{2ah+h^2}}_{\text{moved half the height of the tank}}\ \underbrace{V}_{\text{whole weight}}$