

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC). All integrals once their integrand is simplified may be evaluated with technology exactly or numerically as requested. Otherwise Maple can ONLY be used to check calculations, which should all be done by hand. Always justify limits of integration by mathematical reasoning.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: _____

Date: _____

1. Consider the closed lissajous curve $x = \cos(t)$, $y = \sin(2t)$, $0 \leq t \leq 2\pi$. Because of the obvious symmetry in the 4 quadrants, most of our questions can be answered in the 1st quadrant.

a) Evaluate the area integral $A = \pm \int_t^2 y dx$ in the 1st quadrant to get 1/4 the total area enclosed by the double loop,

exactly and to 4 decimal places. Justify your choice of limits of integration. [You can compare this area roughly to an obvious triangle near the half loop.]

b) Evaluate the slope function $(dy/dx)(t)$ and use it to evaluate the slopes of the two tangent lines at the origin, and then locate the exact coordinates of the point in the 1st quadrant where the tangent line is horizontal and compare the numerical values of the latter with the plot to see if they look right.

2. Consider the curve $x = t^3 - t$, $y = t^3 - 4t$, $t = -2.25 \dots 2.25$.

a) Determine the coordinates of the intercepts with the two axes by solving equations, not by looking at a plot [confirming what you see in the plot]. As t increases, is the curve traced out from left to right or right to left?

b) Evaluate the slope function $(dy/dx)(t)$ at the point where it intersects the positive horizontal axis and compare its numerical value with your plot to see that the number makes sense. [In Maple the 1-1 option and gridlines helps.]

c) Find the coordinates of the point in the first quadrant where the tangent line is vertical, exactly then numerically to compare with the location in your plot to make sure it looks right. Does it? [It helps to plot only the parameter interval where the curve enters and leaves the first quadrant to see the coordinates better in a plot.]

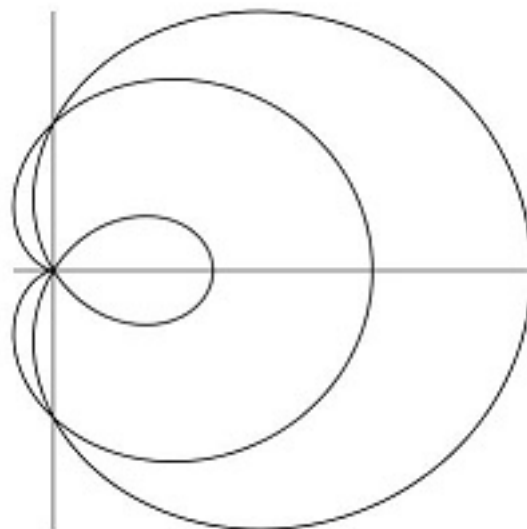
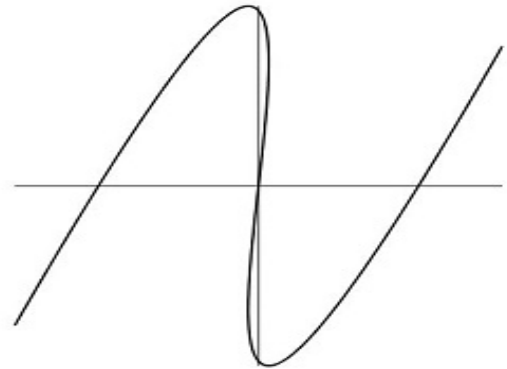
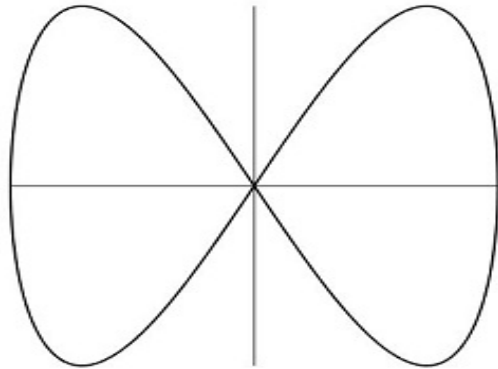
d) Simplify the integrand and evaluate the integral for the arclength of the segment of the curve between the last two horizontal intercepts numerically to 4 decimal places. [Ignore the imaginary 10^{-10} truncation error from the complex expression for the exact value, or first convert to inert form and approximate to 10 digits and then round off to 4 decimal places. You can roughly compare this result to two sides of an approximating right triangle to roughly check your number.]

3. Consider the two cardioids $r = 1 + \cos(\theta)$, $r = 1 + 2 \cos(\theta)$ shown in the figure.

a) Derive the polar and Cartesian coordinates of the two points of intersection away from the origin by solving equations, not by reading off a plot (but then check consistency with your plot!).

b) Simplify the integrand and evaluate the area integral for the area A inside the bigger cardioid and outside the smaller cardioid, both exactly and numerically to 4 decimal places.

e) Evaluate the slopes m of the two tangent lines through the positive y -axis intercept. Evaluate the two angles $\arctan(m)$ these tangent lines make with the horizontal and their difference in degrees to the nearest degree. Do these angles seem to correspond to what you see in the plot?



See figures above. Remember for a single parametrized curve, or a pair of polar coordinate curves, use PlotBuilder or:

```
> plot([t, t^2, t=0..1])
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```
> plot([1, cos(theta)], theta=0..pi, color=[red, blue], coords=polar)
```

Remember: " " (space) or "*" for multiplication ALWAYS, $\pi \neq \text{pi}$, $e \neq \text{e}$. Context menu plots can be copied and pasted together. Gridlines and 1-1 option may be useful.