MAT1505-03/04 19F Take Home Test 3   Print Name (Last, First) _____|___

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

▼ **pledge**

1. Use the Taylor series expansion of e^x about $x = -2$ to approximate $e^{-2.1}$ to three decimal places (i.e., with an error less than $0.5 * 10^{-3}$; use the alternating series estimate). How many terms in the series are needed? Does Maple's direct result agree with your 3 decimal place approximation of the exponential? How many would be needed instead using the Taylor series expansion about $x = 0$? Justify your response.

2. a) Find the (alternating) Taylor series of the standard normal distribution function $N(x) = \dfrac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$ starting from the one for e^x.

b) The probability that a value of the random variable lies within one standard deviation of the mean is

$P = 2\displaystyle\int_0^1 N(x)\ dx$. Use part a) to evaluate P to four decimal places (error less than $0.5 \cdot 10^{-4}$). How many terms are needed for this approximation? Does this agree with Maple's direct evaluation of this integral which Maple can do exactly using the error function?

c) Check the first two nonzero terms of your series for part a) using the Taylor series definition directly.

3. What is the radius of convergence and interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \dfrac{(2x-1)^{2n+1}}{3^{3n} \cdot (n+1)^{\frac{3}{2}}}$? (Be sure to discuss the endpoint behavior.) What does Maple give for its exact and numerical value at the endpoints?

4.The number t of years it takes to double an initial investment at an annual rate of return of r percent satisfies the equation $(1 + 0.01\ r)^t = 2$ whose solution is $t = \dfrac{\ln(2)}{\ln(1 + 0.01\ r)} = f(r)$.

The "rule of 72" states that t is about equal to 72 divided by the rate of growth expressed as a percentage (if the rate is on the order of 10 percent as in the old days), i.e, for an interest rate of 9% it would take 72/9 = 8 years compared to $f(9) \approx 8.04$. It turns out that for smaller interest rates people use the "rule of 70", which works better: a 2% annual rate of return then leads to a doubling time of $\dfrac{70}{2} = 35$ compared to $f(2) \approx 35.00$. These are only "ballpark" estimates so these approximations are good enough and can be evaluated with mental arithmetic.

a) Use the Taylor series approximation $\ln(1 + x) \approx x - \dfrac{x^2}{2} = x\left(1 - \dfrac{x}{2}\right)$ to find the correction factor $1 + R$ to the approximation

$$f(r) \approx \frac{\ln(2)}{0.01 \cdot r \left(1 - \dfrac{0.01\ r}{2}\right)} = \frac{100 \cdot \ln(2)}{r} \cdot (1 + R) \approx \frac{100 \cdot \ln(2)}{r} \approx \frac{69.3}{r} \quad \text{by using the first two terms of}$$

the binomial series for $(1 + x)^{-1}$.

b) Evaluate the numerator $100 \cdot \ln(2) \cdot (1 + R)$ for the two values $r = 2, 9$ to compare with the above stated "rule numbers" 70 and 72. Interesting, no?
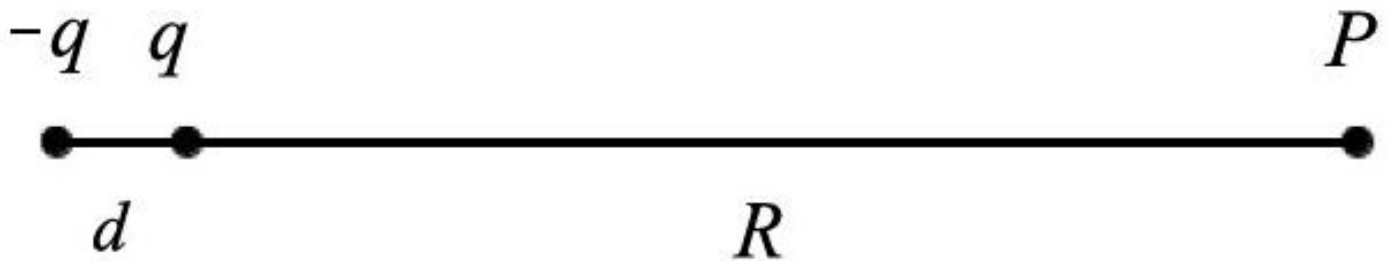
5. The inverse square electric fields of a pair of equal but oppositely signed nearby charges ($\pm q$) cancel out at large distances leaving an inverse cubic "dipole field" as the residual electric field at large distances.

a) Evaluate the lowest order Taylor approximation (the first nonzero term) to its field $E = \dfrac{q}{R^2} - \dfrac{q}{(R + d)^2}$ using the binomial series expansion in the small variable $\dfrac{d}{R} << 1$.

b) What is the fraction of the electric field of a single charge q that this represents, i.e., evaluate the ratio $\dfrac{E}{E_0}$ where $E_0 = \dfrac{q}{R^2}$? Why does this show that your new formula has the same units as the original inverse square forces?

^-q q $\qquad\qquad\qquad\qquad\qquad\qquad P$

$d \qquad\qquad\qquad\qquad\qquad R$

▶ **solution**