MAT1505-03/04 19F Test 2 Answers

① $y = 4(x-1)^{3/2}$

$\frac{dy}{dx} = 4\left(\frac{3}{2}\right)(x-1)^{1/2}(1-0) = 6(x-1)^{1/2}$

$1+\left(\frac{dy}{dx}\right)^2 = 1+36(x-1) = 36x-36+1$
$= 36x-35$

$L = \int_1^2 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_1^2 \sqrt{\underbrace{36x-35}_{u}}\,\underbrace{dx}_{\frac{du}{36}}$

$du = 36dx$

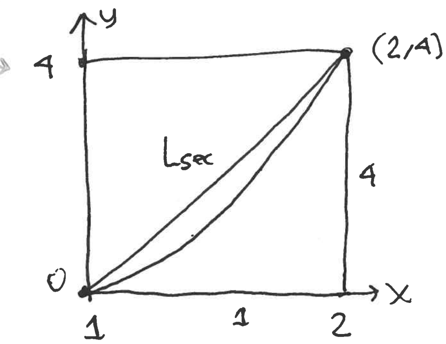
$\int u^{1/2}\frac{du}{36} = \frac{1}{36}\frac{u^{3/2}}{3/2} + C$

$= \boxed{\frac{1}{3\cdot 18}(36x-35)^{3/2}}$ ~~≈ 4.149~~

$= \frac{1}{3\cdot 18}(36x-35)^{3/2}\Big|_1^2$

$= \frac{1}{3\cdot 18}\left[\underbrace{(72-35)}_{37}{}^{3/2} - \underbrace{(36-35)}_{1}{}^{3/2}\right]$

$\boxed{= \frac{1}{54}\left[(37)^{3/2}-1\right] \approx 4.149}$ ≈ 4.149300



$L_{sec} = \sqrt{1+4^2} = \sqrt{17}$

$\approx 4.123 < 4.149$

just a bit less than the curved length, makes sense

② $f(x) = kx^2 \quad 0 \le x \le 1/2$

a) $1 = \int_0^{1/2} kx^2\,dx = \frac{kx^3}{3}\Big|_0^{1/2} = \frac{k}{3}\left(\frac{1}{2}\right)^3 = \frac{k}{24}$

$\boxed{k=24}$, $f(x) = 24x^2$

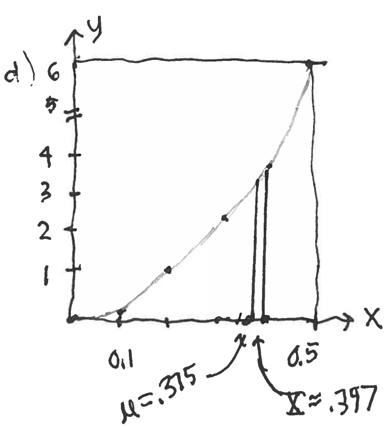
b) $\mu = \int_0^{1/2} x f(x)\,dx = \int_0^{1/2} x(24x^2)\,dx$

$= 24\int_0^{1/2} x^3\,dx = 24\frac{x^4}{4}\Big|_0^{1/2} = 6\left(\frac{1}{2}\right)^4 = \frac{6}{16} = \boxed{\frac{3}{8}}$ ≈ 0.375

c) $\frac{1}{2} = \int_0^{\tilde{x}} 24x^2\,dx = \frac{24x^3}{3}\Big|_0^{\tilde{x}} = 8\tilde{x}^3$

$\tilde{x}^3 = \frac{1}{16}$, $\tilde{x} = \left(\frac{1}{16}\right)^{1/3} = \boxed{\frac{1}{16^{1/3}}} \approx 0.39685$

d)



③ $\int_1^\infty \frac{\arctan x}{x^3}\,dx = \lim_{t\to\infty}\int_1^t \frac{\arctan x}{x^3}\,dx$

$= \lim_{t\to\infty}\left[-\frac{1}{2}\left(\frac{1}{x^2}+1\right)\arctan x - \frac{1}{2x}\right]\Big|_1^t$

$= \lim_{t\to\infty}\left[-\frac{1}{2}\left(\underbrace{\frac{1}{t^2}}_{\to 0}+1\right)\underbrace{\arctan t}_{\to \pi/2} - \underbrace{\frac{1}{2t}}_{\to 0} + \frac{1}{2}(1+1)\underbrace{\arctan 1}_{\pi/4} + \frac{1}{2}\right]$

$= -\frac{1}{2}\left(\frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{1}{2} = -\frac{\pi}{4} + \frac{\pi}{4} + \frac{1}{2} = \boxed{\frac{1}{2}}$