MAT1505-03/04 19F Test 1 Print Name (Last, First) ___________________ , ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

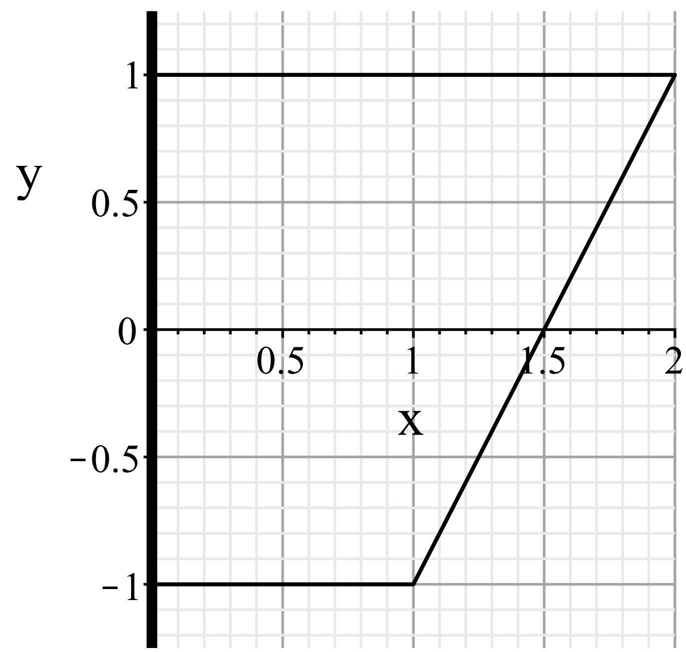
1. Rotate the polygonal region R with vertices $(0,-1)$, $(1,-1)$, $(2,1)$, $(0,1)$ in the x-y plane around the y-axis to obtain a solid of revolution called a frustrum (truncated cone) with base radius $r_1 = 1$ and top radius $r_2 = 2$ and height $h = 2$ from base to top.

a) Write an equation for the slanted line which forms the lateral slanted side of the frustrum when rotated about the vertical axis.

b) Set up an integral representing the volume V of this solid region, and document your integral with an appropriately labeled typical linear cross-section of the region R needed to do this (bullet point endpoints labeled by the starting and stopping value equations of the variable moving along the linear cross-section, shade in with parallel such lines). Evaluate exactly and to 4 decimal places.

c) Suppose a tank has this same solid shape and is filled to the top with a liquid with unit weight density $\rho = 1$ in appropriate but unspecified units, and has a horizontal pipe at the top of the tank where the fluid can exit the tank. Using technology, set up an integral for and evaluate the work done in pumping out the full tank completely to that horizontal connecting pipe height, exactly and to 4 decimal places.

d) Post test learning problem (not today): see reverse side.



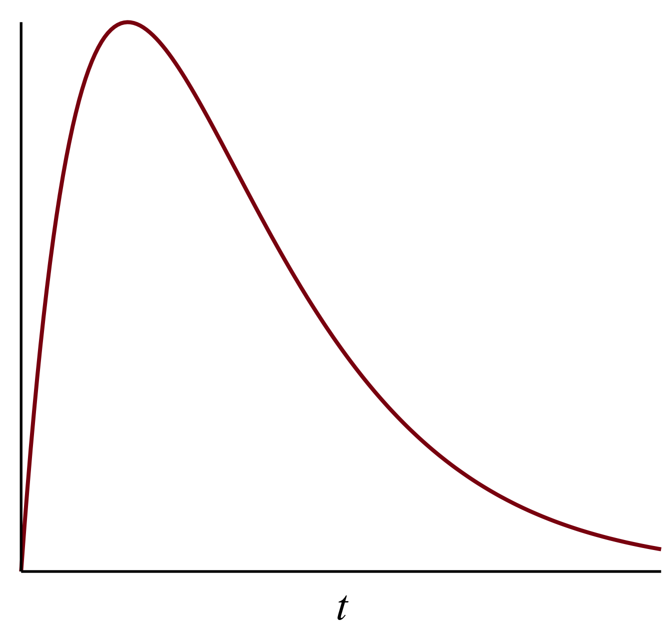
2. Define the prepeak mean lifetime contribution to the lifetime of a radioactive substance undergoing decay as the integral as $M_{prepeak} = k \int_0^{t_p} t\, e^{-kt}\, dt$, where $k > 0$ and $t_p > 0$ is the location of the obvious peak in this integrand, a function which grows initially and then decays away.

a) Determine the value of t_p where the peak occurs.

b) Perform a change of variable to the dimensionless time $T = k\,t$ on this definite integral. Factor out of the integrand any constants which occur.

c) Perform an integration by parts on the remaining integral factor, showing all steps in the process. Box your final answer.

d) Approximate the numerical coefficient to 4 decimal places in this formula.



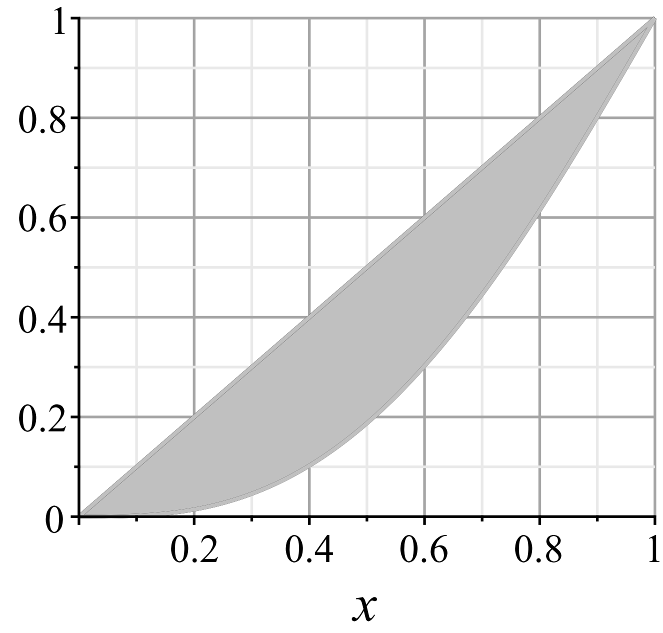
3. A Lorentz curve for $0 \le x \le 1$ is given by the formula $L(x) = x^p \cdot (1 - (1-x)^q) \le x$ whose graph passes through the endpoints $(0, 0)$, $(1, 1)$ of the unit square in the first quadrant where it intersects the curve $y = x$. The Gini index G is twice the area between the Lorentz curve and curve $y = x$ on this interval, which happens to be the ratio of the area between the two curves to the triangular area below the upper curve (namely 1/2 explaining the factor of 2):

$G = 2\ (\textit{Area between graphs})$

a) Evaluate the Gini index when the two exponents have the value $p = 2 = q$ and simplify its formula by expanding out this expression, i.e., first multiply out the integrand by hand to simplify it and then do the integral by hand, then check the original integral with technology. Give the numerical value ot the Gini index to 2 decimal places.

b) What is the average value of the Lorentz curve function on the second half of this interval? Give the exact value and the 3 decimal place approximation.

c) What is the approximate value of x where the Lorentz curve function assumes that average value on the second half of this interval? Give 3 decimal places.



▶ solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

1. d) post test question for those interested in a more serious version of this problem which allows the solution of all these problems with particular parameter values at once:
Suppose the bottom and top radii are now allowed to be free parameters $0 \le r_1 \le r_2$ with top and bottom surfaces at $y = \pm \frac{h}{2}$ with height $h > 0$ to describe a general frustrum shaped tank, i.e., now the vertices in the plane are instead $\left(0, -\frac{h}{2}\right), \left(r_1, -\frac{h}{2}\right), \left(r_2, \frac{h}{2}\right), \left(0, \frac{h}{2}\right)$. Evaluate the work done to empty this tank to a horizontal pipe extending from the top of the tank if it is filled with the same liquid. Start by writing the equation of the line which corresponds to the lateral side of the tank as before (point-slope equation).