MAT2500-01/04 18s Quiz 5 Take Home Print Name (Last, First) ________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC).

1. Suppose Europeans compare the area of our 8.5x11 in standard letter paper size with that of their equivalent A4 dimensions of 21.0x29.7 cm. [Recall 1in = 2.54 cm.] a) Letting $[x, y] = [21.0, 29.7]$ evaluate the differentials of the dimensions of our letter size paper compared with the A4 dimensions, and use the differential approximation to evaluate the approximate change in area and its fractional (and percentage) change. b) Then compare with the exact such changes in the area by evaluating the error in the approximate changes compared to the exact changes (their differences and percentage differences). Remember "US relative to Eur" means US - Eur and (US-Eur)/Eur.

2. a) Find the linear approximation of the function $f(x, y) = 1 - x\,y\cos(\pi y)$ at $(x, y) = (1, 1)$ and use it to approximate $f(1.02, 0.97)$.

b) From your linear approximation, write the simplified equation of the tangent plane and identify the coefficients of the upward normal $\vec{n}$ which follow from that equation.

▶ solution

① a) $(x,y) = (21.0, 29.7)$ $\quad (x+\Delta x, y+\Delta y) = 2.54\,(8.5, 11) = (21.59, 27.94)$

$(\Delta x, \Delta y) = (21.59-21.0,\ 29.7-27.94) = \boxed{(0.59, -1.76) = (dx, dy)}$

$A = xy \quad dA = y\,dx + x\,dy = 29.7(0.59) + 21.0(-1.76) = -19.4370 \approx \boxed{-19.4 = dA}$

$\dfrac{dA}{A} = \dfrac{y\,dx + x\,dy}{xy} = \dfrac{dx}{x} + \dfrac{dy}{y} = \dfrac{-19.437}{623.7} \approx \boxed{-0.0312 = \dfrac{dA}{A}}$ ⤳ US letter size smaller by about 3%

$A = (21.0)(29.7) = 623.7$

b) $A_{USA} = (x+\Delta x)(y+\Delta y) = 603.2246 < A$

$\Delta A = A_{USA} - A = -20.4754 \leftrightarrow dA = -19.437$ close but differ by

$dA - \Delta A = -19.437 + 20.4754 = \boxed{1.0384}$

$\dfrac{dA - \Delta A}{\Delta A} = \dfrac{1.0384}{-20.4754} = -0.05068 \approx \boxed{-0.051}$ ~ about 5% too... what?

dA is 5% less negative than the true difference ΔA.

② a) $f(x,y) = 1 - xy\cos\pi y \qquad f(1,1) = 1 - \cos\pi = 1-(-1) = 2$

$\dfrac{\partial f}{\partial x}(x,y) = -y\cos\pi y \qquad \dfrac{\partial f}{\partial x}(1,1) = -\cos\pi = 1$

$\dfrac{\partial f}{\partial y}(x,y) = -x\dfrac{\partial}{\partial y}(y\cos\pi y) = -x(1\cos\pi y - \pi y\sin\pi y) \qquad \dfrac{\partial f}{\partial y}(1,1) = -(\cos\pi - \pi\sin\pi) = 1$

$L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= 2 + (x-1) + (y-1) \qquad = x + y$

$f(1.02, 0.97) \approx L(1.02, 0.97) = 2 + (1.02-1) + (0.97-1) = 2 + 0.02 - 0.03 = 2 - 0.01 = \boxed{1.99}$

$=$ 1.985009 not a very good approximation, no? maybe not $\frac{.005}{2} = .0025$ only off by 1/4 %

1.990 b) $z = L(x,y) = x + y \rightarrow -x - y + z = 0 \rightarrow \boxed{\vec{n} = \langle -1, -1, 1\rangle}$ $n_3 > 0$ upward normal