MAT2500-01/04 18s Quiz 4 Take Home Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC).

1. The wave heights h in open sea depend on the speed v of the wind and the length of time t that the wind has been blowing at that speed. Values of the function $h = f(v, t)$ are recorded in feet in the table.

(a) What is the value of $f(40, 20)$? What is its meaning in words (a complete sentence)? t
(b) For what value of ~~k~~ t is $f(50, t) = 45$? Formulate this question in words.
(c) What is the meaning of the function $h = f(v, 20)$?

Duration (hours)

v \ t	5	10	15	20	30	40	50
10	2	2	2	2	2	2	2
15	4	4	5	5	5	5	5
20	5	7	8	8	9	9	9
30	9	13	16	17	18	19	19
40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

Wind speed (knots)

(d) Evaluate the average rate of change for $f(v, 20)$ for the intervals $v = 30..40$ and then $v = 40..50$ and then average these to get a decimal value for the "instantaneous" rate of change of $f(v, 20)$ at $v = 40$: $\frac{d}{dv} f(v, 20) \Big|_{v=40}$.

e) Using this result, to what new height would you expect the wave height of 28 ft to decrease if the wind speed decreases from 40 knots to 39 knots at a duration of 20 hours?

▶ solution

① a) $f(40,20) = 28$ When the wind has been blowing at 40 knots for 20 hours, the wave height is 28 ft.

b) $f(50,30) = 45$ so $t = 30$. How long does the wind have to blow at 50 knots for the wave height to be 45 ft?

c) This gives the height of waves as a function of the wind speed when the wind has been blowing for 20 hours.

d)

v \ t	20
30	17
40	28
50	40

$\Delta v = 10$

$\Delta h = 28 - 17 = 11$ $\frac{\Delta h}{\Delta v} = \frac{11}{10} = 1.1$

$\Delta h = 40 - 28 = 12$ $\frac{\Delta h}{\Delta v} = \frac{12}{10} = 1.2$

$\left(\frac{\Delta h}{\Delta v}\right)_{avg} = \frac{1}{2}(1.1 + 1.2) = \frac{1}{2}(2.3) = 1.15$

$\frac{d}{dv} f(v,20) \Big|_{v=40} = \boxed{1.15}$

units : ft/knot

e) $\Delta h = 1.15 \underbrace{\Delta v}_{-1} = -1.15 \longrightarrow h = 28 - 1.15 = \boxed{26.85 \text{ ft}}$