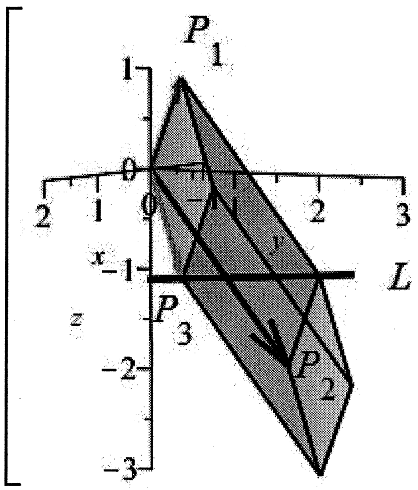
MAT2500-01/04 18S Quiz 2   Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS or arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given three points $P_1(1, 1, 1)$, $P_2(-1, 1, -2)$, $P_3(1, 1, -1)$ and the parallelopiped formed from their three position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3$. [Note $\vec{r}_1, \vec{r}_2, \vec{r}_3$ all point towards us in the first octant, with $\vec{r}_1$ most forward.]



a) Write the parametrized equations of the line L through the points with position vectors $\vec{r}_1 + \vec{r}_2$ and $\vec{r}_3$ as shown in this view, parametrized so that the curve is directed from $\vec{r}_3$ to $\vec{r}_1 + \vec{r}_2$.

b) Find a normal vector $\vec{n}$ for the front face plane $\mathscr{P}_{front}$ of the parallelepiped shown in the figure (passing through P_3)

c) Write the simplified equation for this plane. Does the point at the tip of the parallelopiped ($\vec{r}_1 + \vec{r}_2 + \vec{r}_3$) satisfy this equation as it should?

d) Find the scalar projection h of $\vec{r}_3$ along $\vec{n}$. [$|h|$ is just the distance of the front face plane from the origin, or its height of the parallelopiped with respect to the front face.]

e) Evaluate the area A of the front face of the parallelopiped, a parallelogram formed by the edges parallel to $\vec{r}_1, \vec{r}_2$.

f) Does the volume $V=|h|\,A$ of the parallelopiped equal the triple scalar product $|\vec{r}_3 \cdot (\vec{r}_1 \times \vec{r}_2)|$ as it should?

a) $\vec{r}_1+\vec{r}_2 = \langle 1,1,1\rangle + \langle -1,1,-2\rangle = \langle 1-1, 1+1, 1-2\rangle$
$= \langle 0,2,-1\rangle$

$\vec{a} = \vec{r}_1+\vec{r}_2-\vec{r}_3 = \langle 0,2,-1\rangle - \langle 1,1,-1\rangle = \langle 0-1, 2-1, -1+1\rangle$
$= \langle -1,1,0\rangle$

$\vec{r}_0 = \vec{r}_3$: $\quad \vec{r} = \vec{r}_0 + t\vec{a} = \langle 1,1,-1\rangle + t\langle -1,1,0\rangle$

$\boxed{\langle x,y,z\rangle = \langle 1-t, 1+t, -1\rangle}$

or $\boxed{x = 1-t,\ y = 1+t,\ z = -1}$

b) edges $\vec{r}_1, \vec{r}_2$: $\vec{r}_1 \times \vec{r}_2 = \langle 1,1,1\rangle \times \langle -1,1,-2\rangle$
$\overset{\text{maple}}{=} \boxed{\langle -3,1,2\rangle = \vec{n}}$

c) Choose $\vec{r}_0 = \vec{r}_3 = \langle 1,1,-1\rangle$

$0 = \vec{n}\cdot(\vec{r}-\vec{r}_0) = \langle -3,1,2\rangle \cdot (\langle x,y,z\rangle - \langle 1,1,-1\rangle)$
$= \langle -3,1,2\rangle \cdot \langle x-1, y-1, z+1\rangle$
$= -3(x-1) + (y-1) + 2(z+1) = -3x+y+2z+\underbrace{3-1+2}_{4}$

$\boxed{-3x+y+2z = -4}$

$(\vec{r}_1+\vec{r}_2)+\vec{r}_3 = \langle 0,2,-1\rangle + \langle 1,1,-1\rangle$
$= \langle 0+1, 2+1, -1-1\rangle = \langle 1,3,-2\rangle = \langle x,y,z\rangle$

$-3(1)+(3)+2(-2) = -4$ ✓ check.

d) $|\vec{n}| = \sqrt{9+1+4} = \sqrt{14}$, $\hat{n} = \dfrac{\langle -3,1,2\rangle}{\sqrt{14}}$

$\boxed{h = \hat{n}\cdot\vec{r}_3 = \dfrac{\langle -3,1,2\rangle}{\sqrt{14}} \cdot \langle 1,1,-1\rangle = \dfrac{-3+1-2}{\sqrt{14}} = -\dfrac{4}{\sqrt{14}}} \approx -1.069$

e) $A = |\vec{r}_1 \times \vec{r}_2| = |\vec{n}| = \boxed{\sqrt{14}} \approx 3.742$

f) $V = |h|\,A = \left(\dfrac{4}{\sqrt{14}}\right)(\sqrt{14}) = \boxed{4}$

$\vec{r}_3 \cdot (\vec{r}_1 \times \vec{r}_2) = \vec{r}_3 \cdot \vec{n} = \langle 1,1,-1\rangle \cdot \langle -3,1,2\rangle$
$= 1(-3)+1(1)-1(2) = -3+1-2 = -4$

$V = |\vec{r}_3 \cdot (\vec{r}_1 \times \vec{r}_2)| = \boxed{4}$ agree!

yes!