MAT2500-01/04 18S Final Exam   Print Name (Last, First) ____________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL signs and arrows when appropriate. Always SIMPLIFY expressions. LABEL parts of problem. BOX final short answers. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Use technology to evaluate any integrals you set up. JUSTIFY any numbers or equations that play a role in your calculations.

1. Consider the region R of the plane outside the first circle and inside the second larger circle ***in the fourth quadrant***: $x^2 + (y+2)^2 = 4, x^2 + y^2 = 9, x \ge 0, y \le 0$

a) Find the Cartesian coordinates of their intersection point in this quadrant, and make a diagram of this region shading it in with horizontal linear cross-sections and labeling the three "corner" points with their Cartesian coordinates.

b) Express the three bounding curves in polar coordinates, give the polar coordinates of their intersection points, and include a fully labeled (bullet endpoints with directional arrow midway) typical radial linear cross-section needed to iterate a double integral over R ("shade" the integration region with equispaced radial cross-sections!). What is the polar angle of the intersection point in degrees to the nearest degree?

c) For the vector field $\vec{F} = \langle -y, 2x \rangle$, and the counterclockwise curve C which is the boundary of R, verify Green's Theorem $\int_C \vec{F} \cdot d\vec{r} = \iint_R \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \, dA$ by evaluating both sides of its equation. Use polar coordinates to evaluate the double integral. Once your integrals are set up and simplified, Maple may be used to evaluate them exactly and numerically approximate their common value to 4 decimal places.

2. Consider the snow cone shaped region (solid of revolution) described in Cartesian coordinates by $x^2 + y^2 + (z-2)^2 + z^2 = 4, z^2 = 4x^2 + 4y^2, z \ge 0$.

a) Make an *r-z* half plane diagram of this region, labeling the intersection point of the corresponding curves in this half plane and the axis intercepts of these two cross-section curves in cylindrical coordinates, and shade in the region with equally spaced linear cross-sections with one labeled such cross-section appropriate for a triple integral in these coordinates (choose the appropriate order of integration so a single integral expression is required).

b) Express the bounding surfaces in spherical coordinates, and make a second *r-z* half plane diagram appropriately shaded by linear cross-sections (one labeled at bullet point endpoints) for a triple integral in these coordinates, labeling the intersection point in the *r-z* half plane with its spherical coordinates.

c) Set up a triple integral in cylindrical coordinates for the volume and evaluate it exactly in Maple.

d) Set up a triple integral in spherical coordinates for the volume and evaluate it exactly in Maple.

3. $\vec{\boldsymbol{F}} = \langle 1, \sin(z), y\cos(z) \rangle$.

a) Evaluate the divergence of $\vec{\boldsymbol{F}}$.

b) Show that $\text{curl}(\vec{\boldsymbol{F}}) = \left\langle \frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right\rangle = \vec{\boldsymbol{0}}$, i.e., is a conservative vector field.

c) Find a potential function $f(x, y, z)$ for $\vec{\boldsymbol{F}}$.

d) Evaluate $\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$ along the arc segment $\vec{\boldsymbol{r}} = \langle t, t^2, t^3 \rangle, t = 0..1$. Use Maple to evaluate the integral exactly, then "simplify" the result in Maple.

e) Use the potential to evaluate the line integral $\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$. Do your results for d) and e) agree as they should?

► solution (on-line) turn over to sign pledge!

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in with your answer sheets as a cover page, with the Lastname, FirstName side face up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: