Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Explain in as many words as possible everything you are doing! For each hand integration step, state the antiderivative formula used before

substituting limits into it: $\int_a^b f(x) \ dx = F(x)|_{x=a}^{x=b} = F(b) - F(a)$. Every integral should be checked with Maple. Do

not read off answers from technology unless explicitly requested, otherwise use it only to CHECK your hand calculations. All interated integral calculations must be supported by "cross-section shaded" labeled cross-section diagrams.

1. Consider the solid region R above the x-y plane below the surface $z = x^2 + y^2$ and inside the cylinder $x^2 + y^2 = 2y$.
a) Set up an iterated triple integral for $\iiint_R z \, dV$. [Support you work with a 3d and a 2d diagram.]
b) Evaluate it exactly step by step, using Maple for any nontrivial (nor easy u-sub) antiderivatives, and give its numerical value to 4 decimal places.
c) Check your result with a Cartesian iteration of the same integral, evaluated with Maple.

2. Consider the region of the first quadrant inside the circle $x^2 + y^2 = 4$ but outside the circle $x^2 + y^2 = 2y$.
a) Make a sketch of this region (or print out a plot) and locate a point in the diagram which seems to you to be the geometric center of the region. Annotate it with an arrow to the word "guess". Explain briefly why you made this guess.
b) Set up the 3 iterated integrals in Cartesian coordinates needed to find the centroid, or geometric center of the region, justifying your limits of integration with annotated cross-sectional diagrams, and evaluate the 3 integrals exactly with Maple.
c) Set up the iterated integrals in polar coordinantes needed to find the centroid, or geometric center of the region.
d) Evaluate them exactly in Maple and then approximately to 2 decimal places. Mark your result on the graph, annotated with an arrow to the word "exact". Are you surprised by the comparison with your guess?
e) For a mass distribution ρ which is inversely proportional to the distance from the origin, roughly in which direction from the centroid would you expect to find the center of mass? [Hint: where is there more mass?]
f) Set up the integrals needed to evaluate the center of mass, then evaluate them in Maple exactly and then approximately to 2 decimal places, and mark your result on your graph, annotated with an arrow to the words "center of mass". Does the new point lie in the direction from the centroid you guessed above? Any thoughts if not?

3. Consider the solid region $x^2 + y^2 + z^2 = 4$, $x^2 + y^2 + (z-1)^2 = 1$, $z \geq 0$ inside the larger sphere but outside the smaller sphere.
a) Describe the boundary of this region in cylindrical and spherical coordinates, and make a diagram in the r-z half-plane that illustrates this solid of revolution.
b) Set up the iterated integrals in cylindrical coordinates needed to find the centroid, or geometric center of the region, justifying your limits of integration with annotated cross-sectional r-z halfplane diagram, and evaluate those integrals exactly with Maple. Then evaluate the centroid exactly and numerically to 2 decimal places. [Hint: what does symmetry tell you about the location of the centroid?]
c) Repeat for spherical coordinates.
d) Locate your centroid point on the r-z half-plane diagram. Does it look reasonable for the solid of revolution? Any thoughts about why or why not?

4. Consider the solid region R in the first octant corresponding to the region of integration in the triple integral

$\int_0^1 \int_0^{\sqrt{x}} \int_0^{1-x^2} f(x, y, z) \, dz \, dy \, dx$. See the figures on page 2.

a) Make labeled plane diagrams of the projection of R onto the x-y plane (with a labeled cross-section for the inner integration of the outer double integral) and the x-z plane (with a labeled cross-section for the innermost integral), in each case labeling the cross-section by the starting and stopping value equations for the variable of integration.

b) Rewrite the integral in the order $\iiint \ldots dx\, dz\, dy$, supporting your limits of integration with 2 labeled diagrams as in part a).

c) Evaluate both integrals exactly by hand step by step for $f(x, y, z) = 1$ to get the volume of this region. Your results should agree. [This is not a guarantee that your two interations are correct. If you also get the same result for both iterations for $f(x, y, z) = x$, this would be further evidence.]

► solution

Problem 4 integral region illustrated from two perspectives:
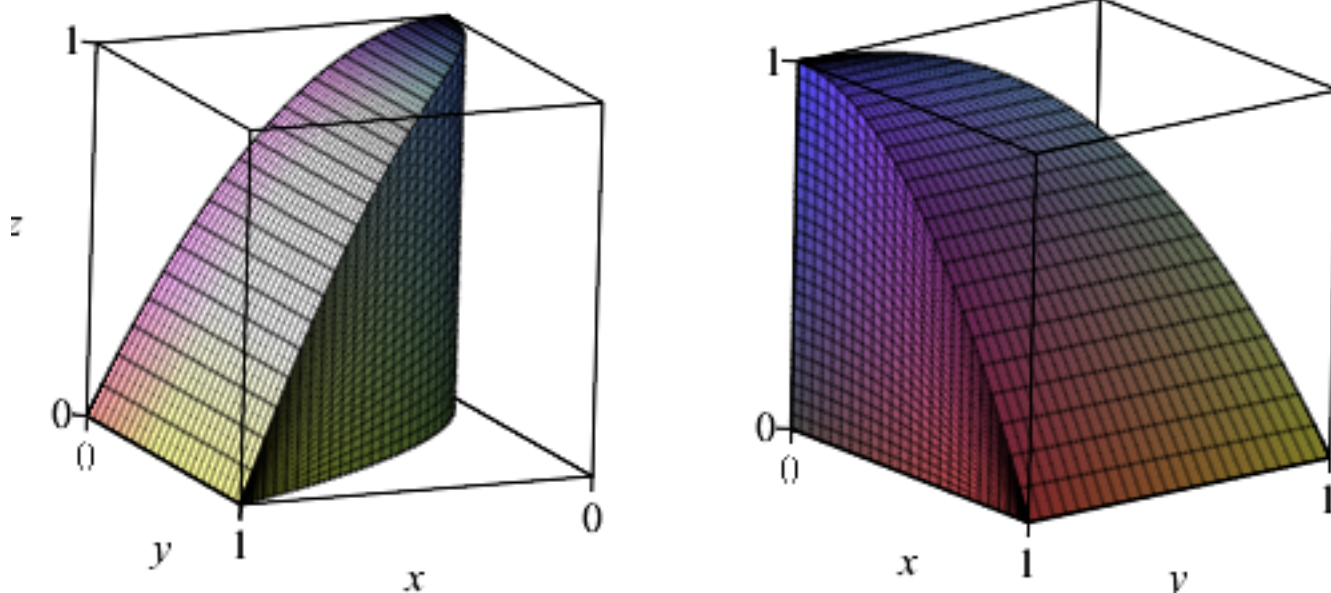
No collaboration. You may only talk to bob. See test rules <u>on-line</u>. Read short rules above. Print out and attach any Maple supporting work you do, hand annotating if necessary with problem number and part etc, whatever is necessary for clarification.

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