

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). You are encouraged to use technology to CHECK but NOT SUBSTITUTE all of your hand results. **Everything on this test is straightforward to evaluate by hand and must be shown explicitly.**

$$1. f(x, y) = x^3 - 3xy + y^3$$

- Evaluate the first and second derivatives of f .
- Find the two critical points of f and the values of f there, and classify them as local extrema or saddle points, justifying your claims. Does a plot (3d or contourplot) confirm your conclusions? Why?
- Find the unit vector \vec{u} along which f increases the most rapidly at the point $(2,0)$. What is the rate of change in that direction? Identify your responses by their proper symbols.
- Evaluate the directional derivative of f at the point $(2,0)$ in the direction of the point $(0,2)$.
- Find a parametrization of the normal line to the graph of f at the point $(2,0,8)$. What are the coordinates of the point where this normal line intersects the x - y plane?

$$2. f(x, y, z) = x^2 + 2y^2 + 3z^2, P(2, -1, 1)$$

- Give the equation of the level surface through P .
- Write an equation for the tangent plane to this level surface at P and simplify the equation of the tangent plane to the simplest standard linear form $ax + by + cz = d$.
- Use the differential approximation to estimate the change in f as one moves from P to the nearby point $(1.95, -0.97, 1.02)$. What is the approximate percentage change?

3. a) Maximize the volume of a rectangular box with base dimensions x and y and height z subject to the constraint $\frac{x}{3} + \frac{y}{2} + z = 1$. What are the dimensions and volume which solve this problem? [No units here and no need to verify the single local maximum which must be the global maximum on physical grounds.

Optional: Explain why.]

b) **Optional.**

How do the intercepts get reflected in the solution? Consider the constraint $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and repeat the problem. Summarize the result you find in words. Is there an aha moment when you reflect on how this result can be understood based on symmetry?

► solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:

"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: _____

Date: _____