MAT2500-01/04 18S Test 1 Answers

a) $\vec r = \langle e^t, e^{-t}, \sqrt2\, t\rangle$

$\vec v = \vec r\,' = \langle e^t, -e^{-t}, \sqrt2\rangle$

$\vec a = \vec r\,'' = \langle e^t, e^{-t}, 0\rangle$

$v = |\vec r\,'| = \sqrt{(e^t)^2+(e^{-t})^2+2} = \sqrt{e^{2t}+2+e^{-2t}}$

$= \sqrt{(e^t+e^{-t})^2} = e^t+e^{-t}$ *perfect square!*

$\hat T = \dfrac{\vec r\,'}{|\vec r\,'|} = \dfrac{\langle e^t, -e^{-t}, \sqrt2\rangle}{e^t+e^{-t}}$

$a = |\vec r\,''| = \sqrt{(e^t)^2+(e^{-t})^2} = \sqrt{e^{2t}+e^{-2t}}$

$e^{\ln 2} = 2$ so:

$\vec r(\ln2) = \langle 2, 1/2, \sqrt2 \ln 2\rangle$

$\vec r\,'(\ln2) = \langle 2, -1/2, \sqrt2\rangle$

$\vec r\,''(\ln2) = \langle 2, 1/2, 0\rangle$

$|\vec r\,'(\ln2)| = 2 + 1/2 = 5/2$

$|\vec r\,''(\ln2)| = \sqrt{4+1/4} = \frac{\sqrt{17}}{2}$

$\hat T(\ln 2) = \frac25\langle 2, -1/2, \sqrt2\rangle = \langle \frac45, -\frac15, \frac{2\sqrt2}{5}\rangle$

b) $\vec r = \underset{\vec r(\ln2)}{\vec r_0} + t\underset{\vec r\,'(\ln 2)}{\vec a}$

$\vec r = \langle 2, 1/2, \sqrt2\ln2\rangle + t\langle 2, -1/2, \sqrt2\rangle$

$\boxed{\langle x,y,z\rangle = \langle 2+2t, \tfrac12-\tfrac12 t, \sqrt2(\ln 2 + t)\rangle}$

c) $\underset{\vec b(t)}{\vec r\,'(t)\times\vec r\,''(t)} = \langle e^t, -e^{-t}, \sqrt2\rangle \times \langle e^t, e^{-t}, 0\rangle$

$\underset{\text{maple}}{=} \boxed{\langle -\sqrt2 e^{-t}, \sqrt2 e^t, 2\rangle}$

$\boxed{\vec b(\ln2) = \langle -\tfrac{\sqrt2}{2}, 2\sqrt2, 2\rangle} = \frac12\underbrace{\langle -\sqrt2, 4\sqrt2, 4\rangle}_{\vec n}$

$|\vec b(t)| = |\vec r\,'(t)\times\vec r\,''(t)| = \sqrt{2e^{-2t}+2e^{2t}+4}$

$= \sqrt2\sqrt{(e^t)^2+2+(e^{-t})^2} = \sqrt2\sqrt{(e^t+e^{-t})^2}$

$= \boxed{\sqrt2(e^t+e^{-t})}$ $\quad |\vec b(\ln 2)| = \sqrt2(2+1/2) = \boxed{\frac{5\sqrt2}{2} = \frac{5}{\sqrt2}}$

f) $\hat B = \dfrac{\langle -\sqrt2 e^{-t}, \sqrt2 e^t, 2\rangle}{\sqrt2(e^t+e^{-t})} = \boxed{\dfrac{\langle -e^{-t}, e^t, \sqrt2\rangle}{e^t+e^{-t}}}$

$\hat B(\ln 2) = \dfrac{\langle -1/2, 2, \sqrt2\rangle}{2+1/2} = \frac25\langle -\frac12, 2, \sqrt2\rangle$

$= \boxed{\dfrac{\langle -1, 4, 2\sqrt2\rangle}{5}}$

g) $\hat N = \hat B\times\hat T = \dfrac{-1}{(e^t+e^{-t})^2}\langle e^t, -e^{-t}, \sqrt2\rangle\times\langle -e^{-t}, e^t, \sqrt2\rangle$

maple $\boxed{\dfrac{\langle \sqrt2, \sqrt2, e^{-t}-e^{t}\rangle}{(e^t+e^{-t})}}$

NOTE: $\hat N(\ln 2) = \dfrac{\langle\sqrt2,\sqrt2, 1/2-2\rangle}{\frac12+2} = \dfrac{\langle \sqrt2, \sqrt2, -3/2\rangle}{5/2} = \frac15\langle 2\sqrt2, 2\sqrt2, -3\rangle$

d) $\underset{2\vec B(\ln 2)}{\vec n}\cdot(\vec r - \underset{r(\ln2)}{\vec r_0}) = 0$

$\langle -\sqrt2, 4\sqrt2, 4\rangle\cdot(\langle x,y,z\rangle - \langle 2, 1/2, \sqrt2\ln2\rangle) = 0$

$-\sqrt2(x-2) + 4\sqrt2(y-1/2) + 4(z-\sqrt2\ln2) = 0$

$-\sqrt2 x + 4\sqrt2 y + 4z + \cancel{2\sqrt2 - 2\sqrt2} - 4\sqrt2\ln 2 = 0$

$\boxed{-\sqrt2\, x + 4\sqrt2\, y + 4z = 4\sqrt2\ln2}$

e) $\kappa = \dfrac{|\vec r\,'\times\vec r\,''|}{|\vec r\,'|^3} = \dfrac{\sqrt2(e^t+e^{-t})}{(e^t+e^{-t})^3} = \boxed{\dfrac{\sqrt2}{(e^t+e^{-t})^2}}$

$\boxed{\rho = \dfrac{(e^t+e^{-t})^2}{\sqrt2}}$ $\quad \rho(\ln 2) = \dfrac{(2+1/2)^2}{\sqrt2} = \boxed{\dfrac{25}{4\sqrt2}}$

f) g) see left column, bottom

h) $a_T(\ln2) = \vec a(\ln 2)\cdot\hat T(\ln 2)$

$= \langle 2, 1/2, 0\rangle\cdot\langle 2, -1/2, \sqrt2\rangle(\frac25)$

$= (4-\frac14)\frac25 = \frac{15}{4}\cdot\frac25 = \boxed{\frac32}$

$a_N(\ln 2) = \vec a(\ln2)\cdot\hat N(\ln 2)$

$= \langle 2, 1/2, 0\rangle\cdot\dfrac{\langle\sqrt2, \sqrt2, 1/2-2\rangle}{5/2}$

$= (2\sqrt2 + \frac12\sqrt2)\cdot\frac25 = \frac52\sqrt2(\frac25) = \boxed{\sqrt2}$

[NOTE $a_T^2 + a_N^2 = \frac94 + 2 = \frac{17}{4} = a^2$ ✓]

i) $L = \int_{-2}^2 v\,dt = \int_{-2}^2 e^t + e^{-t}\,dt$

$= e^t - e^{-t}\Big|_{-2}^2$

$= (e^2-e^{-2}) - (e^{-2}-e^2)$

$= \boxed{2(e^2-e^{-2}) \approx 14.5074}$

j) $\vec C = \vec r + \rho\vec N \xrightarrow{t=\ln 2}$

$\vec C(\ln 2) = \langle 2, 1/2, \sqrt2\ln2\rangle + \underbrace{\left(\frac{25}{4\sqrt2}\right)\frac15\langle 2\sqrt2, 2\sqrt2, -3\rangle}_{\frac54\langle 2, 2, -3/\sqrt2\rangle}$

$= \langle 2+\frac{10}{4}, \frac12+\frac{10}{4}, \sqrt2\ln2 - \frac{15}{4\sqrt2}\rangle$

$= \langle \frac92, 3, \underbrace{\sqrt2(\ln 2 - \tfrac{15}{8})}\rangle$

$\quad\; = 4.5$ ✓, ✓, $\approx -1.67139 \approx -1.67$ ✓

Confession: I can be a bit sloppy doing my own test!