

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Maple can only be used to check not justify responses.

1. $3y'' - 4y' - 4y = 0, y(0) = 2, y'(0) = -4$ [independent variable: x].
 - a) Find a basis $\{y_1, y_2\}$ of the solution space.
 - b) Use that basis to express the general solution $y(x)$ of the DE.
 - c) Using 2×2 matrix methods involving the inverse matrix, find the solution which satisfies the initial conditions, showing all work (report the value of the inverse matrix you use).
 - d) Show that this IVP solution actually solves the DE by backsubstitution into that DE.
 - e) What are the two characteristic lengths $0 < \tau_1 < \tau_2$ for the two exponentials in this problem?
 - f) Use technology to plot your result for $x = -2..2$ and an appropriate vertical window and make a rough sketch of what you see, labeling the axes with variable names and key tickmarks on your sketch. (Clicking on the gridlines icon for a Maple plot helps you make your hand sketch more accurate.)
 - g) Use calculus to determine **exactly** (rules of exponents and logs! simplify your result for y to a single simple term) the x and y values of the point of inflection on the graph and then their approximate values to 4 decimal places. Locate on your sketch and label the point of inflection which should be roughly apparent in your plot. Do the numbers you found agree with what your eyes see in the technology plot? [Yes or no, with an explanation would be a good response.]

► **solution**

① a) $y = e^{rx}$
 $y' = re^{rx}$
 $y'' = r^2 e^{rx}$

$$3r^2 e^{rx} - 4r e^{rx} - 4e^{rx} = 0$$

$$(3r^2 - 4r - 4) e^{rx} = 0$$

$$3r^2 - 4r - 4 = 0$$

$$r = \frac{4 \pm \sqrt{16 - 4(3)(-4)}}{2 \cdot 3}$$

$$= \frac{4 \pm \sqrt{16 + 48}}{6} = \frac{4 \pm \sqrt{64}}{6} = \frac{4 \pm 8}{6}$$

$$= 2, -\frac{2}{3}$$

Just use Maple here!

$y = e^{2x}, e^{-\frac{2}{3}x} \rightarrow \{e^{2x}, e^{-\frac{2}{3}x}\}$ is basis

b) $y = c_1 e^{2x} + c_2 e^{-\frac{2}{3}x}$

c) $y' = 2c_1 e^{2x} - \frac{2}{3}c_2 e^{-\frac{2}{3}x}$

$$y(0) = c_1 + c_2 = 2$$

$$y'(0) = 2c_1 - \frac{2}{3}c_2 = -4$$

$$\begin{bmatrix} 1 & 1 \\ 2 & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -\frac{2}{3} \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ -4 \end{bmatrix} = \frac{1}{-\frac{2}{3} - 2} \begin{bmatrix} -\frac{2}{3} & -1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

a) continued

$$= \frac{3}{8} \begin{bmatrix} \frac{2}{3} & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -4 \end{bmatrix} = \frac{3}{8} \begin{bmatrix} \frac{4}{3} - 4 \\ 4 + 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$y = -e^{2x} + 3e^{-\frac{2}{3}x}$

d) $r = \frac{1}{2}, \frac{2}{3} \rightarrow \tau_1 = \frac{1}{2} < \tau_2 = \frac{3}{2}$

f)

g) $y' = -2e^{2x} - 2e^{-\frac{2}{3}x}$
 $[y'' = -4e^{2x} + \frac{4}{3}e^{-\frac{2}{3}x} = 0] \frac{3}{4} e^{\frac{2}{3}x}$
 $-3e^{\frac{8}{3}x} + 1 = 0$
 $e^{\frac{8}{3}x} = \frac{1}{3}$
 $x = -\frac{3}{8} \ln 3$
 $y = -e^{2(-\frac{3}{8} \ln 3)} + 3e^{-\frac{2}{3}(-\frac{3}{8} \ln 3)}$
 $= -3^{-3/4} + 3 \cdot 3^{1/4} = (-\frac{3}{8} \ln 3, \frac{9}{3} 3^{1/4}) \approx (-0.4120, 3.51096)$

Just use Maple!

Boxed Answer: $(x, y) = (-\frac{3}{8} \ln 3, \frac{9}{3} 3^{1/4}) \approx (-0.4120, 3.51096)$