MAT2705-04/05 18F Quiz 5 Print Name (Last, First) ____________________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $2\,x_1 + 4\,x_2 + -6\,x_3 - x_4 = 6$

$x_1 + 2\,x_2 - 3\,x_3 = 4$

$4\,x_1 + 8\,x_2 - 12\,x_3 - 4\,x_4 = 8$

a) Write down the coefficient matrix **A**, the RHS matrix $\vec{\mathbf{b}}$ and the augmented matrix $\mathbf{C} = \langle \mathbf{A} \mid \vec{\mathbf{b}} \rangle$ for this linear system of equations.

b) With technology (identify your choice!), reduce this matrix **C** step by step to its ReducedRowEchelonForm avoiding fractions, recording the intermediate matrices and row operations for each step (as in $R_1 \leftrightarrow R_2$, $R_3 \to R_3 + 2\,R_1$, $R_1 \to \frac{1}{2}\,R_1$). You may combine the AddRow operations within a single pivot, reporting only the final matrix.

c) Write out the equations that correspond to the reduced matrix. Identify the leading variables (L) and the free variables (F) and solve. State your solution in the scalar form: $x_1 = \ldots$, $x_2 = \ldots$, etc , then right in column matrix form and identify the coefficient vectors of your free variable parameters.

d) Enter the augmented matrix into Maple and by right context panel menu, find the reduced matrix and the solution of the system of equations. Write down exactly what Maple gives you for the column matrix solution and compare with your reduced matrix and solution. They should agree. Do they?

▶ solution

① $\underbrace{\begin{bmatrix}2&4&-6&-1\\1&2&-3&0\\4&8&-12&-4\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix}6\\4\\8\end{bmatrix}}_{\vec{b}} \to C = \begin{bmatrix}2&4&-6&-1&6\\1&2&-3&0&4\\4&8&-12&-4&8\end{bmatrix}$

could divide by 4 here too

b) $\xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix}1&2&-3&0&4\\2&4&-6&-1&6\\4&8&-12&-4&8\end{bmatrix} \xrightarrow[R_3 \to R_3 - 4R_1]{R_2 \to R_2 - 2R_1}$

$\begin{bmatrix}1&2&-3&0&4\\0&0&0&-1&-2\\0&0&0&-4&-8\end{bmatrix} \xrightarrow{R_2 \to -R_2} \begin{bmatrix}1&2&-3&0&4\\0&0&0&1&2\\0&0&0&-4&-8\end{bmatrix} \xrightarrow{R_3 \to R_3 + 4R_2}$

$$\begin{array}{ccccc} L & F & F & L & \\ x_1 & x_2 & x_3 & x_4 & \end{array}$$
$$\begin{bmatrix}1&2&-3&0&4\\0&0&0&1&2\\0&0&0&0&0\end{bmatrix}$$

c) $x_1 + 2x_2 - 3x_3 = 4$

$x_4 = 2$

$0 = 0$

$\xrightarrow{\;x_2 = t_1,\ x_3 = t_2\;}$ $x_1 = 4 - 2t_1 + 3t_2$, $x_4 = 2$

soln: $\boxed{x_1 = 4 - 2t_1 + 3t_2,\ x_2 = t_1,\ x_3 = t_2,\ x_4 = 2}$ $\to$ $\boxed{\vec{x} = \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}4-2t_1+3t_2\\t_1\\t_2\\2\end{bmatrix}} = \begin{bmatrix}4\\0\\0\\2\end{bmatrix} + t_1\underbrace{\begin{bmatrix}-2\\1\\0\\0\end{bmatrix}}_{\vec{u}_1} + t_2\underbrace{\begin{bmatrix}3\\0\\1\\0\end{bmatrix}}_{\vec{u}_2}$

$\vec{u}_1, \vec{u}_2$ are the coefficient vectors of the free variable parameters.

d) Maple confirms this reduced matrix and gives the RHS solution vector

$\begin{bmatrix}4 - 2\_t_2 + 3\_t_3\\ \_t_2\\ \_t_3\\ 2\end{bmatrix}$ corresponding to our $t_1 = \_t_2$ (from x_2), $t_2 = \_t_3$ (from x_3).