MAT2705-04/05 18F Quiz 4 Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations if appropriate for interpretation). Indicate where technology is used and what type (Maple, GC). Always justify claims.

a) Solve this initial value problem for the velocity $v \geq 0$ of a body moving horizontally in a medium whose resistance is proportional to the three halves power of the velocity:

$$\frac{dv}{dt} = -k\,v^{\frac{3}{2}},\ v(0) = v_0 .$$

b) Does your form of the solution permit the obvious solution $v = 0$? Can you rewrite it to do so? If so, do it.

c) What is the limiting velocity as $t \to \infty$?

d) **OPTIONAL.** Integrate your velocity function $v(t)$ to get the position function $x(t)$ for which $x(0) = 0$, either by hand or using Maple, or equivalently use Maple to solve the initial value problem $x'(t) = v(t), x(0) = 0$.

Show that your result can be expressed in the form $x(t) = \frac{2\sqrt{v_0}}{k} \cdot \left(1 - \frac{2}{2 + k\,t\sqrt{v_0}}\right)$

from which it follows that the total finite displacement as $t \to \infty$ is $x_\infty = \frac{2\sqrt{v_0}}{k}$.

a) $\frac{dv}{dt} = -kV^{3/2} \xrightarrow[\text{arate}]{\text{sep}} \int V^{-3/2}dV = \int -k\,dt \rightarrow \frac{V^{-1/2}}{-1/2} = -kt + C_1 \rightarrow V^{-1/2} = \frac{1}{2}kt \underbrace{-\frac{1}{2}C_1}_{+C}$

$V^{-1/2} = \frac{1}{2}kt + C \rightarrow V = (\frac{1}{2}kt + C)^{-2}$ gen soln.

$\downarrow t=0$

$V_0^{-1/2} = 0 + C \rightarrow C = V_0^{-1/2}$ $\rightarrow$ $\boxed{V = (\frac{1}{2}kt + V_0^{-1/2})^{-2}}$ (Note $V \geq 0$, $V_0 \geq 0$)

b) $= [(\frac{1}{2}ktV_0^{1/2} + 1)(V_0^{-1/2})]^{-2} = \boxed{\frac{V_0}{(1 + \frac{1}{2}ktV_0^{1/2})^2}}$

this allows solution $V_0 = 0 \rightarrow V = 0$.

c) $V_\infty = \lim_{t\to\infty} \frac{V_0}{\underbrace{(1 + \frac{1}{2}ktV_0^{1/2})^2}_{\to\infty!}} = \boxed{0}$

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d) $X = \int (\frac{1}{2}kt + V_0^{-1/2})^{-2}\,dt = \int u^{-2}(\frac{2}{k}du) = \frac{2}{k}\frac{u^{-1}}{-1} + C_2 = -\frac{2}{k}(\frac{1}{2}kt + V_0^{-1/2})^{-1} + C_2$

$\left(\begin{array}{l} u = \frac{1}{2}kt + V_0^{-1/2} \\ du = \frac{1}{2}k\,dt \end{array}\right)$

for fun

$0 = X(0) = -\frac{2}{k}(V_0^{-1/2})^{-1} + C_2 = -\frac{2V_0^{1/2}}{k} + C_2 \rightarrow C_2 = \frac{2V_0^{1/2}}{k}$

$X = \frac{2V_0^{1/2}}{k} - \frac{2}{k(\frac{1}{2}kt + V_0^{-1/2})} = \frac{2V_0^{1/2}}{k} - \frac{2V_0^{1/2}}{k(\frac{1}{2}ktV_0^{1/2} + 1)} = \frac{2V_0^{1/2}}{k} - \frac{4V_0^{1/2}}{k(ktV_0^{1/2} + 2)}$

$= \frac{2V_0^{1/2}}{k}\left(1 - \frac{2}{ktV_0^{1/2} + 2}\right)$

Maple gives this form of the soln of the IVP

$X_\infty = \lim_{t\to\infty} X = \frac{2V_0^{1/2}}{k}(1 - 0) = \frac{2V_0^{1/2}}{k}$