MAT2705-04/05 18F Test 2 Answers

1 a)

$$\left.\begin{aligned} 3x_1 + x_2 - 3x_3 &= 6 \\ 2x_1 + 7x_2 + x_3 &= -9 \\ 2x_1 + 5x_2 \qquad &= -5 \end{aligned}\right\}$$ linear system of eqns in scalar form

$$\underbrace{\begin{bmatrix} 3 & 1 & -3 \\ 2 & 7 & 1 \\ 2 & 5 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 6 \\ -9 \\ -5 \end{bmatrix}}_{\vec{b}}$$ matrix form of these equations is a matrix eqn!

$$A^{-1} = \begin{bmatrix} 5 & 15 & -22 \\ -2 & -6 & 9 \\ 4 & 13 & -19 \end{bmatrix}$$

$$\vec{x} = A^{-1}\vec{b} = \begin{bmatrix} 5 & 15 & -22 \\ -2 & -6 & 9 \\ 4 & 13 & -19 \end{bmatrix} \begin{bmatrix} 6 \\ -9 \\ -5 \end{bmatrix}$$ matrix soln

$$= \begin{bmatrix} 5(6) - 15(9) + 5(22) \\ -2(6) + 6(9) - 9(5) \\ 4(6) - 13(9) + 19(5) \end{bmatrix} \overset{\text{Maple!}}{=} \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix}$$

show this to prove you understand matrix multiplication

$$\boxed{x_1 = 5,\ x_2 = -3,\ x_3 = 2}$$

b) $\det A \overset{\text{maple}}{=} -1 \neq 0$

so A^{-1} exists which guarantees unique soln.

2 a)

$$\begin{bmatrix} 3 & 1 & -3 & 11 & 10 & 0 \\ 5 & 8 & 2 & -2 & 7 & 0 \\ 2 & 5 & 0 & -1 & 14 & 0 \end{bmatrix} = \langle A | \vec{0} \rangle \text{ if } A\vec{x} = \vec{0}$$

$$\xrightarrow[\text{rref}]{\text{maple}} \begin{array}{c} \begin{matrix} L & L & L & F & F & \\ x_1 & x_2 & x_3 & x_4 & x_5 & \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 2 & -3 & 0 \\ 0 & 1 & 0 & -1 & 4 & 0 \\ 0 & 0 & 1 & -2 & -5 & 0 \end{bmatrix} \end{array}$$

$$\begin{aligned} x_1 + 2x_4 - 3x_5 &= 0 \to x_1 = -2t_1 + 3t_2 \\ x_2 - x_4 + 4x_5 &= 0 \to x_2 = t_1 - 4t_2 \\ x_3 - 2x_4 \ \ 5x_5 &= 0 \to x_3 = 2t_1 + 5t_2 \end{aligned}$$

F: $x_4 = t_1,\ x_5 = t_2$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2t_1 + 3t_2 \\ t_1 - 4t_2 \\ 2t_1 - 5t_2 \\ t_1 \\ t_2 \end{bmatrix} = t_1 \begin{bmatrix} -2 \\ 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} 3 \\ -4 \\ 5 \\ 0 \\ 1 \end{bmatrix}}$$

call them $\vec{u}_1, \vec{u}_2$ — basis vectors of soln space: dimension $= \boxed{2}$

point of confusion

soln space $\longrightarrow$ span$\{\vec{u}_1, \vec{u}_2\}$ = 2d subspace of $\mathbb{R}^5$ (coefficient space)

at most 3 of the 5 vectors in $\mathbb{R}^3$ are linearly independent, so that span$\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5\} = \mathbb{R}^3$

in fact $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ are a basis of $\mathbb{R}^3$

b)

$\vec{u}_1$: $-2\vec{v}_1 + \vec{v}_2 + 2\vec{v}_3 + \vec{v}_4 = \vec{0}$

$\vec{u}_2$: $3\vec{v}_1 - 4\vec{v}_2 + 5\vec{v}_3 + \vec{v}_5 = \vec{0} \xrightarrow[\text{solve}]{\text{c)}} \boxed{\vec{v}_5 = -3\vec{v}_1 + 4\vec{v}_2 - 5\vec{v}_3}$

leading columns of A

$5 - 2 = \boxed{3 \text{ lin ind vectors}}$

3 b)

$$y_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + y_2 \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$$

unknowns here are the new coords! appropriate symbols are y_1, y_2

$$\begin{bmatrix} 1 & -3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 9 \\ 5 \end{bmatrix} = \frac{1}{2+9} \begin{bmatrix} 2 & 3 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ 5 \end{bmatrix}$$

$$= \frac{1}{11} \begin{bmatrix} 2(9) + 3(5) \\ -3(9) + 1(5) \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 18 + 15 \\ -27 + 5 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 33 \\ -22 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} \checkmark$$

c)

$$\vec{v}_4 = -\vec{v}_1 + 2\vec{v}_2 = -\begin{bmatrix} 1 \\ 3 \end{bmatrix} + 2 \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 - 6 \\ -3 + 4 \end{bmatrix} = \boxed{\begin{bmatrix} -7 \\ 1 \end{bmatrix}}$$

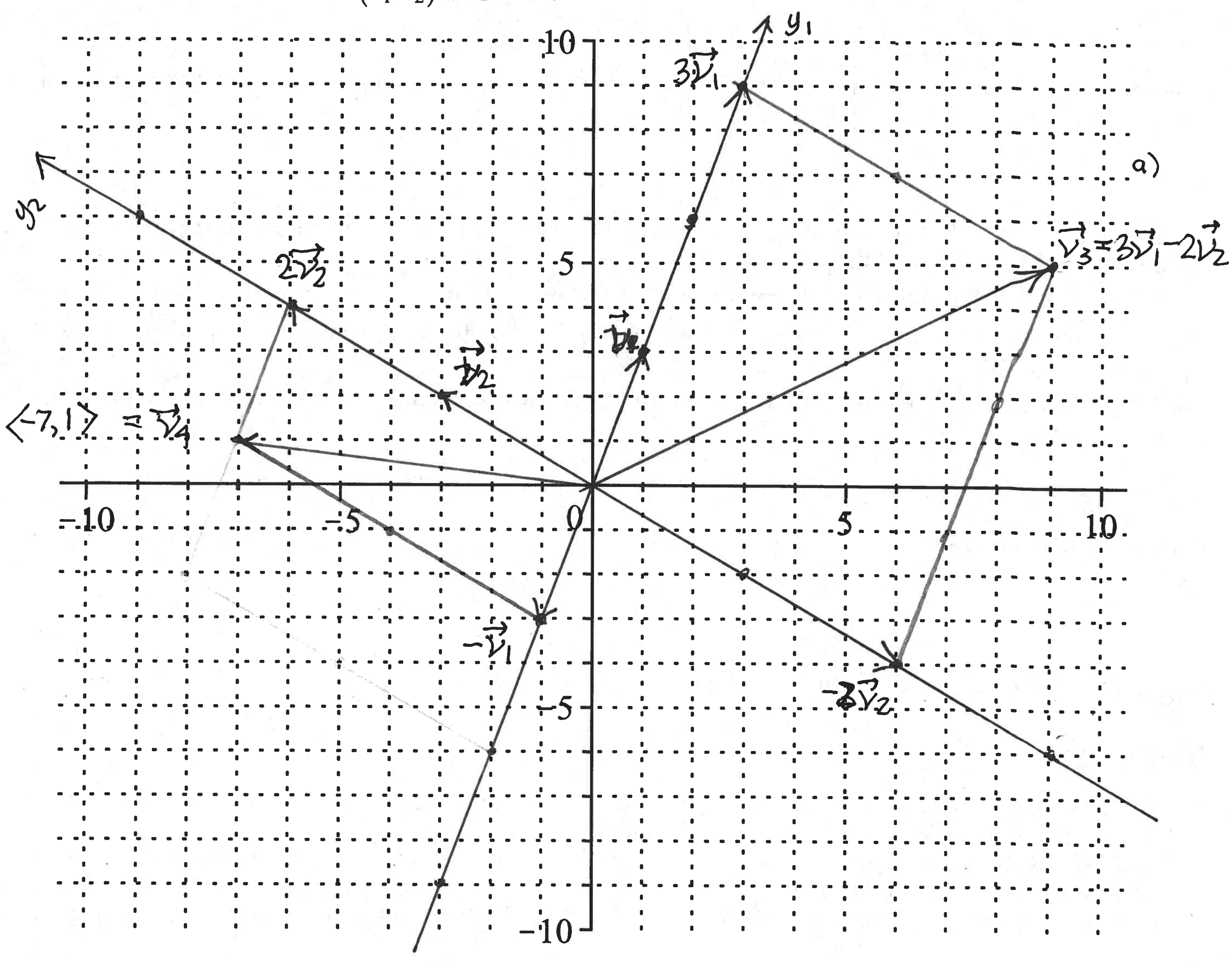
agrees with graph!

WARNING: you cannot multiple a column matrix (vector) on the right by a square matrix!

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

2x1 2x2 dont match, NO GO!

3. a) On the grid below, **draw in** arrows representing the vectors $\vec{v}_1 = \langle 1, 3\rangle$, $\vec{v}_2 = \langle -3, 2\rangle$ and $\vec{v}_3 = \langle 9, 5\rangle$ and **label** them by their symbols. **Extend** the basis vectors $\{\vec{v}_1, \vec{v}_2\}$ to the corresponding coordinate axes for (y_1, y_2) and **mark** the positive direction with an arrow head and the axis label. Mark off tickmarks on these axes for integer values of the new coordinates.Then **draw in** the parallelogram with edges parallel to the new axes for which $\vec{v}_3$ is the main diagonal and shade it in in pencil lightly. Read off the coordinates (y_1, y_2) of $\vec{v}_3$ with respect to these two vectors (write them down) and **express** $\vec{v}_3$ as a linear combination of these vectors; **put this equation** at the tip of this vector.
b) Now use matrix methods to express $\vec{v}_3$ as a linear combination of the other two vectors (show all steps in this process), box it and then check your linear combination by expanding it out.
c) **Draw in** the arrow representing the vector $\vec{v}_4$ whose new coordinates are $(y_1, y_2) = (-1, 2)$ and **label** the tip of $\vec{v}_4$ by its symbol. Draw in the projection parallelogram associated with the new coordinates and lightly shade it in in pencil. Determine its old coordinates (x_1, x_2) graphically.



▶ **solution**