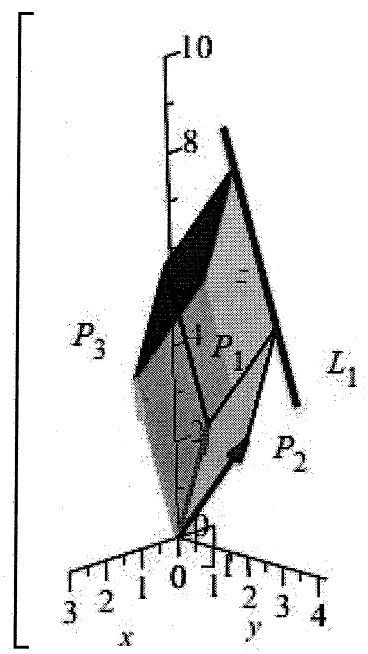
MAT2500-01/04 17S Quiz 2   Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given three points $P_1(1,2,3)$, $P_2(-1,1,2)$, $P_3(2,1,4)$ and the parallelopiped formed from their three position vectors $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$. [Note $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$ all point towards us in the first octant, with $\vec{r}_1$ most forward.]



a) Write the parametrized equations of the line L_1 through the $\vec{r}_1 + \vec{r}_2$ as shown in this view. Where does this line intersect the $z=0$ plane?

b) Find a normal vector $\vec{n}$ for the plane $\mathcal{P}_{topface}$ which contains the top back left face (shaded) of the parallelepiped shown in the figure.

c) Write the simplified equation for this plane. Does the point at the tip of the parallelopiped (where the line L_1 touches the top left back face $\mathcal{P}_{topface}$) ~~satisfy this equation as it should?~~ okay

d) Find the scalar projection h of $\vec{r}_3$ along $\vec{n}$. [$|h|$ is just the distance of the top face plane from the origin, or its height if we instead think of that face as the top of the parallelopiped.]

e) Evaluate the area A of the top left back face of the parallelopiped, a parallelogram formed by the edges parallel to $\vec{r}_1$, $\vec{r}_2$.

f) Does the volume $V=|h|\,A$ of the parallelopiped equal the triple scalar product $|\vec{r}_3 \cdot (\vec{r}_1 \times \vec{r}_2)|$ as it should?

a) $\vec{r}_1 = \langle 1,2,3\rangle$ $\vec{r}_1+\vec{r}_2 = \langle 1-1, 2+1, 3+2\rangle = \langle 0,3,5\rangle = \vec{r}_0$

$\vec{r}_2 = \langle -1,1,2\rangle$

$\vec{r}_3 = \langle 2,1,4\rangle$

L_1 is parallel to $\vec{r}_3 = \vec{a}$

$\vec{r} = \vec{r}_0 + t\vec{a} = \langle 0,3,5\rangle + t\langle 2,1,4\rangle$

$\langle x,y,z\rangle = \langle 2t, 3+t, 5+4t\rangle$ vector equation

$\boxed{x=2t,\ y=3+t,\ z=5+4t}$ scalar equations

$0=z \to t=-5/4 \to x=-5/2,\ y=3-5/4=7/4$, so at pt $\boxed{(-5/2, 7/4, 0)}$

b) edges clearly equal to $\vec{r}_1$ and $\vec{r}_2$ as difference vectors between endpoints so

$\vec{n} = \vec{r}_1\times\vec{r}_2 = \langle 1,2,3\rangle \times \langle -1,1,2\rangle \overset{\text{Maple}}{=} \boxed{\langle 1,-5,3\rangle}$

c) plane passes through P_3 so $\vec{r}_0 = \vec{r}_3 = \langle 2,1,4\rangle$

$0 = \vec{n}\cdot(\vec{r}-\vec{r}_0) = \langle 1,-5,3\rangle \cdot \langle x-2, y-1, z-4\rangle$

$= (x-2) - 5(y-1) + 3(z-4) = x-5y+3z\underbrace{-2+5-12}_{-9}$

$\boxed{x-5y+3z=9}$

$\vec{r}_1+\vec{r}_2+\vec{r}_3 = \langle 0,3,5\rangle + \langle 2,1,4\rangle = \langle 2,4,9\rangle \to 2-5(4)+3(9) = 2+27-20 = 2+7 = 9$ ✓ yes!

d) $|\vec{n}| = \sqrt{1+25+9} = \sqrt{35}$, $\hat{n} = \dfrac{\langle 1,-5,3\rangle}{\sqrt{35}}$

$h = \hat{n}\cdot\vec{r}_3 = \dfrac{\langle 1,-5,3\rangle\cdot\langle 2,1,4\rangle}{\sqrt{35}} = \dfrac{2-5+12}{\sqrt{35}} = \boxed{\dfrac{9}{\sqrt{35}} = \dfrac{9}{35}\sqrt{35}}$

e) $\vec{r}_1\times\vec{r}_2 = \langle 1,-5,3\rangle$ from b)

$A = |\vec{r}_1\times\vec{r}_2| = \boxed{\sqrt{35}}$ from d).

f) $V = |h|A = \left(\dfrac{9}{\sqrt{35}}\right)\sqrt{35} = 9$

$\vec{r}_3\cdot(\vec{r}_1\times\vec{r}_2) = \vec{r}_3\cdot\vec{n} = \langle 2,1,4\rangle\cdot\langle 1,-5,3\rangle$

$= 2-5+12 = 9$

$|\vec{r}_3\cdot(\vec{r}_1\times\vec{r}_2)| = 9$ ✓   yes, they agree.