Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL signs and arrows when appropriate. Always SIMPLIFY expressions. LABEL parts of problem. BOX final short answers. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Use technology to evaluate any integrals you set up. JUSTIFY any numbers that play a role in your calculations.

1. Consider the region R of the plane inside the circle between the two straight lines:
$(x-1)^2+y^2=1,\ 16\,y^2=9\,x^2$
a) Find the three intersection points and draw a diagram illustrating this region, "shading it in" by equally spaced radial cross-sections of the region, and labeling all key points and tickmarks.
b) Express the three bounding curves in polar coordinates, give the polar coordinates of the intersection points, and include a fully labeled (bullet endpoints with directional arrow midway) typical radial cross-section needed to iterate a double integral over R.
c) For the vector field $\vec{F} = \langle x-y, x\rangle$, verify Green's Theorem $\int_C \vec{F}\cdot d\vec{r} = \iint_R \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\, dA$

by evaluating both sides of its equation. Use polar coordinates to evaluate the double integral. Evaluate the contribution along the circle using y as the parameter (**optional:** check it with another parametrization using the polar angle θ). Once your integrals are set up and simplified, Maple may be used to evaluate them exactly and numerically approximate them to 4 decimal places.

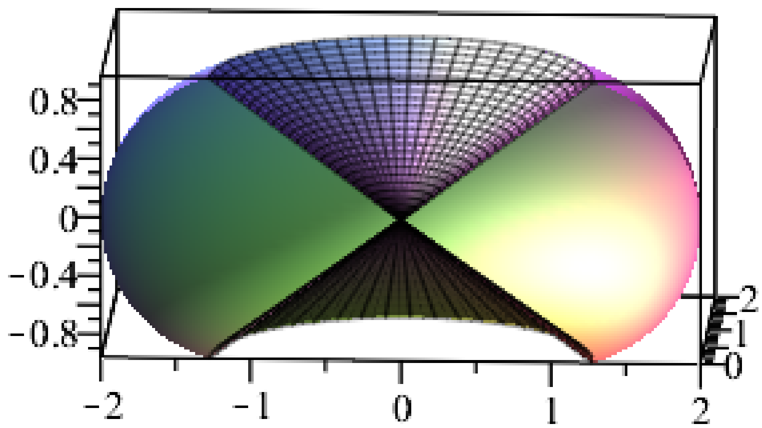
2. Consider the equatorial wedge cut out of the torus of revolution between the two "nappes" of the cone described in cylindrical coordinates by $(r-1)^2+z^2=1,\ 16\,z^2=9\,r^2$ illustrated on page 2 of this exam with a cutaway view of half the region.
a) Make an r-z half plane diagram of this region, labeling the intersection points and the axis intercepts of these two cross-section curves.
b) Express the bounding surfaces in spherical coordinates, and label the intersection points in the r-z half plane with their spherical coordinates.
c) Set up a triple integral in spherical coordinates for the volume and evaluate it exactly in Maple.

3. $\vec{\boldsymbol{F}} = \langle 36\,x+9\,y+6\,z,\ 9\,y+9\,x+3\,z,\ 4\,z+3\,y+6\,x\rangle$.
a) Evluate the divergence of $\vec{\boldsymbol{F}}$.
b) Show that $\operatorname{curl}(\vec{\boldsymbol{F}}) = \left\langle \frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right\rangle = \vec{\boldsymbol{0}}$, i.e., is a conservative vector field.
c) Verify that $f(x,y,z) = 18\,x^2 + \frac{9}{2}\,y^2 + 2\,z^2 + 9\,x\,y + 3\,y\,z + 6\,x\,z$ is a potential function for $\vec{\boldsymbol{F}}$.

d) Evaluate $\int_C \vec{\boldsymbol{F}}\cdot d\vec{\boldsymbol{r}}$ along the arc segment $\vec{\boldsymbol{r}} = \langle \sin(t), \sin(t), \cos(t)\rangle,\ t=0..\frac{\pi}{2}$. [You will need Maple for the antiderivatives of powers of the trig functions, show the integration steps.]

e) Use the potential to evaluate the line integral $\int_C \vec{\boldsymbol{F}}\cdot d\vec{\boldsymbol{r}}$ over any curve between the same endpoints with the same orientation. Do your results for c) and d) agree as they should?

► solution (on-line) turn over to sign pledge!



pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in with your answer sheets as a cover page, with the Lastname, FirstName side face up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: