MAT2500-01/04 17S Takehome Test 3 Print Name (Last, First) _____|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Explain in as many words as possible everything you are doing! For each hand integration step, state the antiderivative formula used before substituting limits into it: $\int_a^b f(x)\ dx = F(x)|_{x=a}^{x=b} = F(b) - F(a)$. Every integral should be checked with Maple. Do not read off answers from technology unless explicitly requested, otherwise use it only to CHECK your hand calculations.

1. $\int_1^2 \int_0^{8x-4x^2} x\, dy\, dx$ a) Evaluate this exactly by hand step by step (easy!) and by technology.

b) Make a completely labeled diagram of the region of integration with a typical labeled cross-section representing the current iteration of the integral.
c) Make a NEW completely labeled diagram corresponding to the reversed order of integration.
d) State the new integral with the order of integration reversed.
e) Evaluate the new integral by exactly by hand (you need a simple *u*-substitution) and using technology.
f) Do you get the same result as in part a)?

2. $\int_{-1}^{0} \int_{\sqrt{2-x^2}}^{1+\sqrt{1-x^2}} x\, y\, dy\, dx$.

a) Make a completely labeled (shaded by typical cross-sections) diagram of the region of integration for this integral, with a typical correctly labeled cross-section line segment (bullet endpoints, arrowhead) representing the current iteration of the integral.
b) Make a NEW diagram appropriate for the evaluation of this integral in polar coordinates.
c) Convert the integral to polar coordinates and simplify.
d) Evaluate this integral by hand step by step, easily done by hand without technology antiderivatives.

3. Consider the solid region *R* in the first corresponding to the triple integral $\int_0^2 \int_{x^2}^4 \int_0^{\frac{y-x^2}{2}} 1\, dz\, dy\, dx$

See the figures illustrating *R* on page 2.
a) What equations describe the 4 faces of this solid? Describe each face (top, bottom, back, side, etc) with its equation.
b) Re-iterate this triple integral in the order dx dz *dy*, showing how you "deconstruct" it to understand the region *R*. Support your new limits of integration with a diagram for the outer double integral with completely labeled line segment cross-sections and equally spaced such cross-sections for the shading, and a 3d diagram for the innermost integral indicating one typical completely labeled linear cross-section.
c) Check your two integrals exactly using technology, reporting Maple's results. They should agree with each other. Do they?

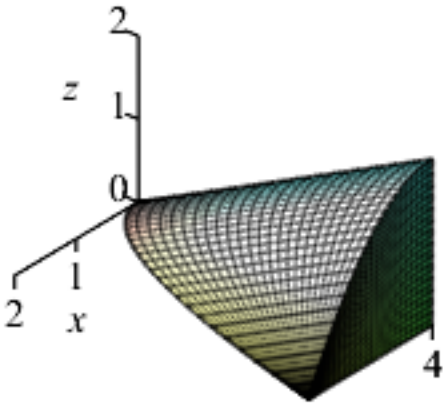
4. Consider the solid region *R* below the sphere $(z-2)^2 + x^2 + y^2 = 4$ and above the cone $z^2 = 4(x^2 + y^2)$ **in the first octant**.
a) Express these conditions first in cylindrical coordinates and then in spherical coordinates, using appropriate symbols for those coordinates.
b) Using cylindrical coordinates make a diagram of the corresponding two bounding curves in the *r*-*z* half-plane and find the values of (r, z) of their point of intersection (representing the circle at which the two surfaces intersect) . Shade in the region corresponding to *R* with vertical cross-section lines and label the endpoints of a typical one by the starting and stopping value equations for z .
c) Write down a triple integral in cylindrical coordinates for the volume $V = \iiint 1\ dV$ of *R* and for the triple

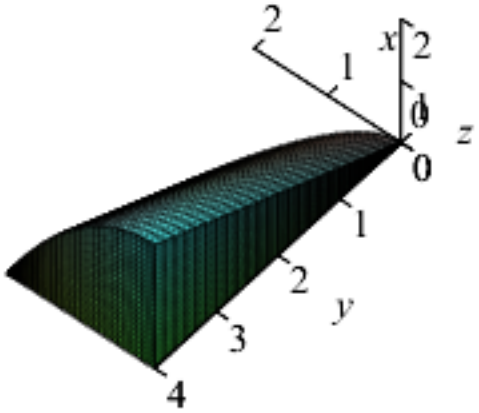
integral $Vz = \iiint z\, dV$ of z over this region. Evaluate them exactly using Maple.

d) Next express the same two integrals in spherical coordinates and evaluate those integrals step by step, confirming the results given by Maple. Support your work with a completely labeled typical cross-section shading of the region in a NEW diagram.

e) Evaluate numerically to 2 decimal places their ratio giving the z value of the centroid. It should seem reasonable. Does it? Why?

Problem 3 integral region illustrated from two perspectives:





solution (on-line)

No collaboration. You may only talk to bob. See test rules on-line. Read short rules above. Print out and attach any Maple supporting work you do, hand annotating if necessary with problem number and part etc, whatever is necessary for clarification.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in stapled to your answer sheets as the cover page, with the Lastname, FirstName side face up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: