

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). You are encouraged to use technology to CHECK but NOT SUBSTITUTE all of your hand results, but **everything on this test is straightforward to evaluate by hand and must be shown explicitly.**

1. $f(x, y) = x y e^{-x-y}$; $P(2, -2)$

- Evaluate the gradient of f and its value at P .
- Find a unit vector in the direction in which f increases most rapidly at P . What is its rate of change in this direction?
- Evaluate the directional derivative of f at P in the direction towards the point $(5, 2)$.
- Write an equation for the level curve through P and its tangent line (as a linear condition on (x, y)).
- Write an equation for the linear approximation to f at P and simplify the equation of the tangent plane to the graph $z = f(x, y)$ above P to the standard linear form $a x + b y + c z = d$.

- Evaluate the differential of the function $Y = k N P e^{-N-P}$, where $k > 0$ is a constant and $N > 0$, $P > 0$.
- What is the single critical point of this function? What is its value there?
- Classify this critical point using the second derivative test.

3. Minimize the area $A = x y + 2 z (x + y)$ of an open topped box with base dimensions x and y and height z subject to the constraint of constant volume $V = x y z = 32$. What are the dimensions and volume which solve this problem? [No units here and no need to verify the local minimum.][Oops, should have asked for the area.]

► solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:
 "During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: _____

Date: _____