MAT2500-01/04 17S Test 1 Answers

a) $\vec{r} = \langle \cos t, 2\sin t, t\rangle$ $\qquad \vec{r}(\frac{\pi}{2}) = \langle 0, 2, \pi/2\rangle$

$\vec{v} = \vec{r}\,' = \langle -\sin t, 2\cos t, 1\rangle$ $\qquad \vec{r}\,'(\frac{\pi}{2}) = \langle -1, 0, 1\rangle$

$\vec{a} = \vec{r}\,'' = \langle -\cos t, -2\sin t, 0\rangle$ $\qquad \vec{r}\,''(\frac{\pi}{2}) = \langle 0, -2, 0\rangle$

$v = |\vec{r}\,'| = \sqrt{\underbrace{\sin^2 t}_{1-\cos^2 t} + 4\cos^2 t + 1}$ $\qquad |\vec{r}\,'(\frac{\pi}{2})| = \sqrt{2}$

$= \sqrt{2 + 3\cos^2 t}$ $\qquad \hat{T}(\frac{\pi}{2}) = \dfrac{\langle -1, 0, 1\rangle}{\sqrt{2}}$

$\hat{T} = \dfrac{\vec{r}\,'}{|\vec{r}\,'|} = \dfrac{\langle -\sin t, 2\cos t, 1\rangle}{\sqrt{2+3\cos^2 t}}$ $\qquad |\vec{r}\,''(\frac{\pi}{2})| = 2$

$a = |\vec{r}\,''| = \sqrt{\cos^2 t + 4\underbrace{\sin^2 t}_{1-\cos^2 t}}$

$= \sqrt{4 - 3\cos^2 t}$

The vector parametrized equation of a line is a relationship between $\vec{r} = \langle x, y, z\rangle$ and t.
The scalar parametrized eqns of a line is a relationship between x, y, z and t.

b) $\vec{r} = \vec{r}(\frac{\pi}{2}) + s\,\vec{r}\,'(\frac{\pi}{2}) = \langle 0, 2, \pi/2\rangle + s\langle -1, 0, 1\rangle$

$$\boxed{\begin{array}{l}\langle x, y, z\rangle = \langle -s, 2, \frac{\pi}{2} + s\rangle \\ \text{or } x = -s,\ y = 2,\ z = \pi/2 + s\end{array}}$$

c) $\vec{b}(t) = \langle -s, 2c, 1\rangle \times \langle -c, -2s, 0\rangle$

Maple $\boxed{\langle 2\sin t, -\cos t, 2\rangle}$ $\qquad \boxed{\vec{b}(\frac{\pi}{2}) = \langle 2, 0, 2\rangle \quad |\vec{b}(\frac{\pi}{2})| = 2\sqrt{2}}$

$|\vec{b}(t)| = \sqrt{\underbrace{4\sin^2 t}_{4-4\cos^2 t} + \cos^2 t + 4}$

$= \boxed{\sqrt{8 - 3\cos^2 t}}$

d) $\vec{b}(\frac{\pi}{2}) \cdot (\vec{r} - \vec{r}(\frac{\pi}{2})) = 0$

$\langle 2, 0, 2\rangle \cdot \langle x - 0, y - 2, z - \frac{\pi}{2}\rangle = 0$

$2(x) + 2(z - \frac{\pi}{2}) = 0$

$\boxed{x + z = \frac{\pi}{2}}$

e) $\kappa = \dfrac{|\vec{b}|}{|\vec{r}\,'|^3} = \boxed{\dfrac{\sqrt{8 - 3\cos^2 t}}{(2 + 3\cos^2 t)^{3/2}}}$

$\rho = \boxed{\dfrac{(2 + 3\cos^2 t)^{3/2}}{(8 - 3\cos^2 t)^{1/2}}}$

$\rho(\frac{\pi}{2}) = \dfrac{2^{3/2}}{8^{1/2}} = \dfrac{2^{3/2}}{2^{3/2}} = \boxed{1}$

f) $\hat{B} = \dfrac{\vec{b}}{|\vec{b}|} = \dfrac{\langle 2\sin t, -\cos t, 2\rangle}{\sqrt{8 - 3\cos^2 t}}$

$\hat{B}(\frac{\pi}{2}) = \dfrac{\langle 2, 0, 2\rangle}{2^{3/2}} = \boxed{\dfrac{\langle 1, 0, 1\rangle}{\sqrt{2}}}$

g) $\hat{N} = \hat{B} \times \hat{T} = \dfrac{\langle 2s, -c, 2\rangle \times \langle -s, 2c, 1\rangle}{\sqrt{8 - 3c^2}\,\sqrt{2 + 3\cos^2 t}}$

$= \boxed{\dfrac{\langle -5\cos t, -4\sin t, 3\sin t\cos t\rangle}{\sqrt{8 - 3\cos^2 t}\,\sqrt{2 + 3\cos^2 t}}}$

$\hat{N}(\frac{\pi}{2}) = \dfrac{\langle 0, -4, 0\rangle}{\sqrt{8(2)}} = \boxed{\langle 0, -1, 0\rangle}$

h) $a_T(\frac{\pi}{2}) = \hat{T}(\frac{\pi}{2}) \cdot \vec{a}(\frac{\pi}{2})$

$= \dfrac{\langle -1, 0, 1\rangle}{\sqrt{2}} \cdot \langle 0, -2, 0\rangle = \boxed{0}$

$a_N(\frac{\pi}{2}) = \hat{N}(\frac{\pi}{2}) \cdot \vec{a}(\frac{\pi}{2})$

$= \langle 0, -1, 0\rangle \cdot \langle 0, -2, 0\rangle = \boxed{2}$

i) $s = \int_0^{2\pi} |\vec{r}\,'(t)|\,dt$

$= \int_0^{2\pi} \sqrt{2 + 3\cos^2 t}\,dt$

$= \boxed{4\sqrt{5}\,\text{EllipticE}(\sqrt{15}/5)}$

$\approx \boxed{11.6135}$

j) $\vec{C}(\frac{\pi}{2}) = \vec{r}(\frac{\pi}{2}) + \rho(\frac{\pi}{2})\hat{N}(\frac{\pi}{2})$

$= \langle 0, 2, \frac{\pi}{2}\rangle + 1\langle 0, -1, 0\rangle = \langle 0, 1, \pi/2\rangle$

k) $\vec{Osc}(\frac{\pi}{2}) = \langle 0, 1, \frac{\pi}{2}\rangle + \cos\theta\,\dfrac{\langle -1, 0, 1\rangle}{\sqrt{2}} + \sin\theta\,\langle 0, -1, 0\rangle$

$\vec{r}(\frac{\pi}{2})$ is 1 unit in y-direction from center so other end of diameter is 1 unit in opposite direction:

$\langle 0, 1, \frac{\pi}{2}\rangle - \langle 0, 1, 0\rangle = \langle 0, 0, \pi/2\rangle$

in fact at $\theta = \pi$, circle just reaches ~~z~~-axis at $z = \pi/2$.