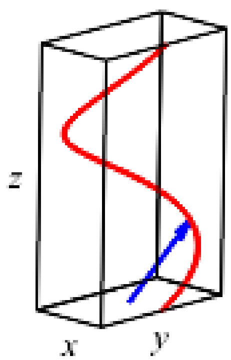
MAT2500-01/04 17S Test 1 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). You are encouraged to use technology to check all of your hand results.



The parametrized curve segment
$\vec{r}(t) = \langle \cos(t), 2\sin(t), t \rangle, 0 \le t \le 2\pi$
is shown in the figure together with $\vec{r}\left(\frac{\pi}{2}\right)$. Maple will automatically simplify most results to expressions only involving the cosine, or you can ask it to simplify them.

a) Evaluate and simplify $\vec{v}(t) = \vec{r}\,'(t)$, $\vec{a}(t) = \vec{r}\,''(t)$, $v(t) = |\vec{r}\,'(t)|$, $\vec{T}(t)$, $a(t) = |\vec{r}\,''(t)|$ and their values (including $\vec{r}(t)$) at $t = \frac{\pi}{2}$.

b) Write the parametrized equations of the tangent line through $\vec{r}\left(\frac{\pi}{2}\right)$.

c) Evaluate and simplify $\vec{b}(t) = \vec{r}\,'(t) \times \vec{r}\,''(t)$. Evaluate $\vec{b}\left(\frac{\pi}{2}\right)$ and $\left|\vec{b}\left(\frac{\pi}{2}\right)\right|$.

d) Write the equation of the osculating plane through $\vec{r}\left(\frac{\pi}{2}\right)$ containing the tangent vector and the second derivative there.

e) Evaluate the curvature $\kappa(t) = \dfrac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$ and its reciprocal, the radius of curvature $\rho(t)$ and $\rho\left(\frac{\pi}{2}\right)$.

f) Evaluate and simplify the unit vector $\vec{B}(t)$ in the direction of $\vec{b}(t) = \vec{r}\,'(t) \times \vec{r}\,''(t)$ and then $\vec{B}\left(\frac{\pi}{2}\right)$.

g) Evaluate and simplify the unit normal $\vec{N}(t) = \vec{B}(t) \times \vec{T}(t)$.

h) Evaluate the scalar tangential projection $a_T\left(\frac{\pi}{2}\right)$ along $\vec{T}\left(\frac{\pi}{2}\right)$ of the acceleration $\vec{a}\left(\frac{\pi}{2}\right) = \vec{r}\,''\left(\frac{\pi}{2}\right)$ and its scalar normal projection $a_N\left(\frac{\pi}{2}\right) = \vec{N}\left(\frac{\pi}{2}\right) \cdot \vec{a}\left(\frac{\pi}{2}\right)$ exactly.

i) Write down an integral formula for the length of the curve $\vec{r}(t)$ for $t = 0 .. 2\pi$. Evaluate it exactly (special function!) and then numerically to 4 decimal places.

j) **Optional.** The center of the osculating circle has position vector: $\vec{C}(t) = \vec{r}(t) + \rho(t)\,\vec{N}(t)$. Show that $\vec{C}\left(\frac{\pi}{2}\right) = \left\langle 0, 1, \frac{\pi}{2} \right\rangle$.

k) **Optional.** [Just kidding. Ignore this until after the test.] The osculating circle is given by $\overrightarrow{Osc}(t, \theta) = \vec{C}(t) + \rho(t)\left(\cos(\theta)\vec{T}(t) + \sin(\theta)\,\vec{N}(t)\right)$. Evaluate this for $t = \frac{\pi}{2}$. Then show that this circle just kisses the z-axis. This is obvious in a plot or from thinking about the relative position of $\vec{r}\left(\frac{\pi}{2}\right)$ and $\vec{C}\left(\frac{\pi}{2}\right)$.

solution

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: