MAT1505-03/04 17F Quiz 7 Print Name (Last, First) ________________, ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. $a_n = \arctan\left(\frac{n^2}{4+n^2}\right)$.

a) Show the evaluation of $A = \lim_{n\to\infty} a_n$.

b) Evaluate the decimal values of the first 20 terms on your favorite tech device, but don't bother listing them on this sheet. Do these seem to confirm part a)? If so, at which term in the sequence (give $n$ and $a_n$ to 5 decimal places) does the term get within 1 percent of the limiting value? If not, reconsider part a).

2. Find the sum of the series exactly and to 4 decimal places: $S = \sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$. Support your work by identifying the values of the two parameters needed for the relevant formula.

3. $\sum_{n=3}^{\infty} \frac{1}{n^5 - 5n^3 + 4n}$. Use Maple to find a formula for the $n$th partial sum of this infinite series, and then take its limit to obtain the value of this series, and finally compare your exact result with Maple's value for the infinite series.

## solution

① a) $\lim_{n\to\infty} \arctan\left(\frac{n^2}{4+n^2}\right) = \lim_{n\to\infty} \arctan\left(\frac{n^2/n^2}{(4+n^2)/n^2}\right) = \lim_{n\to\infty} \arctan\left(\frac{1}{\underbrace{\frac{4}{n^2}}_{\to 0}+1}\right)$

$= \arctan 1 = A$ (since $\lim_{n\to\infty} \frac{4}{n^2} = 0$, obvious)

$= \boxed{\frac{\pi}{4}} \approx 0.7854$ (agrees with Maple limit)

b) This is an increasing sequence so to get within 1% of A it must become larger than $.99A \approx 0.77754$

$a_{15} = 0.77658$, $\boxed{a_{16} = 0.77765}$ so is the first such term.

② $S = \sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}} = \sum_{n=1}^{\infty} \frac{(-3)(-3)^n}{(2^3)^n} = \sum_{n=1}^{\infty} (-3)\left(-\frac{3}{8}\right)^n$ geometric series

ratio $\boxed{r = -\frac{3}{8}}$

first term $n=1$: $\frac{(-3)^2}{2^3} = \boxed{\frac{9}{8} = a}$ $\quad S = \frac{a}{1-r} = \frac{9/8}{1-(-3/8)} = \frac{9/8}{11/8} = \boxed{\frac{9}{11}} \boxed{\approx 0.8181}$

③ $S_N = \sum_{n=3}^{N} \frac{1}{n^5 - 5n^3 + 4n} = \boxed{-\frac{1}{24(N-1)} + \frac{1}{8N} - \frac{1}{8(N+1)} + \frac{1}{24(N+2)} + \frac{1}{96}} \xrightarrow[N\to\infty]{\lim} = \boxed{\frac{1}{96}}$ same as Maple

$\overset{\text{simplify}}{=} \frac{N^4 + 2N^3 - N^2 - 2N - 24}{96N^4 + 192N^3 - 96N^2 - 192N}$

$= \frac{1 + 2/N - 1/N^2 - 2/N^3 - 24/N^3}{96 + 192/N - 96/N^2 - 192/N^3} \xrightarrow[N\to\infty]{\lim} \boxed{\frac{1}{96}}$ since negative powers of N go to zero

Maple: $\sum_{n=1}^{\infty} \frac{1}{n^5 - 5n^3 + 4n} = \boxed{\frac{1}{96}}$