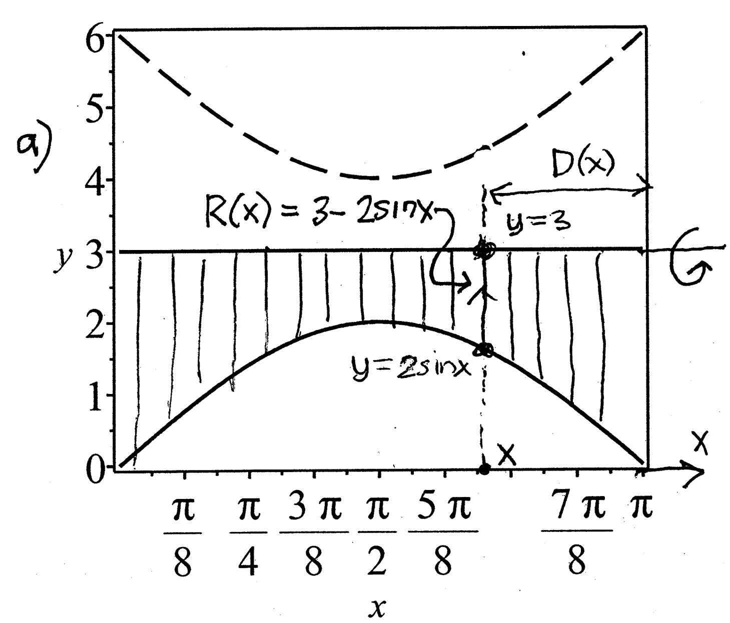
MAT1505-03/04 17F Quiz 3 Print Name (Last, First) ______________, ______________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

The region R is bounded by the curves $y = 2\sin(x)$ and $y = 3$ over the interval $0 \le x \le \pi$ is shown to the right.
a) Shade in only R with equally spaced vertical cross-sections, and identify the radius $R(x)$ on a typical such cross-section, labeling its bullet endpoints with the starting and stopping value equations.
b) Rotate this region around the axis $y = 3$ to form an hourglass solid region on its side. Write down an integral V_1 (with simplified integrand) representing the volume of the resulting solid of revolution.
c) Evaluate this integral exactly using Maple and then approximate the result to 4 decimal places.
d) Write down an integral representing the work needed to move each layer of this solid at position x from its present position to the right face by a constant horizontal force of 1 force unit per unit volume (like unit gravity, setting $\rho\, g = 1$ as if the x-axis were measuring height above the ground,



i.e., $W = \int_a^b A(x)\, \mathrm{D}(x)\, dx$, where $A(x)$ is the cross-sectional area and $\mathrm{D}(x)$ is the displacement each layer at position x undergoes).
e) Evaluate this integral exactly using Maple and then approximate the result to 4 decimal places.
d) If you concentrate all the weight at the obvious center of the 3d solid region of revolution, the midpoint of the rotation axis, we could move it to the "top" a distance of half the axis length. Compare that work done this way (volume times half axis displacement) with the result of part e). Interesting, no?

▶ **solution**

b) $V = \int_0^\pi \underbrace{A(x)}_{\pi R(x)^2}\, dx = \boxed{\int_0^\pi \pi(3-2\sin x)^2\, dx} \overset{c)}{\underset{\text{Maple}}{=}} \boxed{11\pi^2 - 24\pi} \approx 33.16742473$

$\approx \boxed{33.1674}$

$R(x) = 3 - 2\sin x$

$\left(= \int_0^\pi \pi(9 - 12\sin x + \underbrace{4\sin^2 x}_{\text{need tech. for antiderivative}}\, dx\right)$

d) $W = \int_0^\pi A(x)\underbrace{D(x)}_{\pi - x}\, dx = \boxed{\int_0^\pi \pi(3-2\sin x)^2(\pi - x)\, dx} \overset{e)}{\underset{\text{Maple}}{=}} \boxed{\frac{11}{2}\pi^3 - 12\pi^2} \approx 52.0992690$

$\approx \boxed{52.0993}$

$\pi - x$ ← "variable" displacement! otherwise we don't need to integrate!

f) $W_{cow} = V \cdot \frac{\pi}{2} = (11\pi^2 - 24\pi)\frac{\pi}{2} = \boxed{\frac{11}{2}\pi^3 - 12\pi^2 = W}$!

"center of weight"

They are equal!
This is interesting to bob, at least.
[This is the power of symmetry.]