MAT1505-03/04 17f Quiz 1 Print Name (Last, First) ________________ , ______________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. The fractional volume v of liquid contained in the bottom of a hemispherical tank up to a fractional height $\hbar$ (of the full radius height for a full tank) is given by the formula $\frac{dv}{d\hbar} = \frac{3}{2}\cdot(2\hbar - \hbar^2)$. What is the change in the fractional volume when the fractional height increases from $\hbar = 0.20$ to $\hbar = 0.30$?

[**Optional challenge** for after the quiz and after covering volumes of revolution: derive this rate of change formula.]

$\int_{V_1}^{V_2} dV = V_2 - V_1 = \Delta V$ definite integral $\Leftrightarrow$ net change theorem!

$\int dV = V + C$ indefinite integral

2. $\displaystyle\int_0^{20} 60000 \frac{e^{-0.6t}}{(1+5e^{-0.6t})^2}\, dt$.

a) Use the change of variable method to rewrite this integral as a new definite integral in the variable $u = 1 + 5e^{-0.6t}$. (with new limits of integration!).

b) Evaluate the integral by hand.

c) Compare the result you find with the technology evaluation of the original integral.

▶ solution

① $\Delta V = \int_{0.2}^{0.3} \frac{dV}{dh}\, dh = \int_{0.2}^{0.3} \frac{3}{2}(2h - h^2)\, dh = \frac{3}{2}\left(h^2 - h^3/3\right)\Big|_{0.2}^{0.3}$

standard notation for change in V

$= \frac{3}{2}\left[(0.3^2 - 0.3^3/3) - (0.2^2 - 0.2^3/3)\right] \overset{\text{Maple}}{=} \boxed{0.0655}$

② a) $\displaystyle\int_0^{20} \frac{60000\, e^{-0.6t}\, dt}{(\underbrace{1+5e^{-0.6t}}_{u})^2} = \int_{t=0}^{t=20} 60000\left(-\tfrac{1}{3}\right)\frac{du}{u^2} = \boxed{-\int_6^{1.000030721} 20000\, u^{-2}\, du}$

$u = 1 + 5e^{-0.6t}$

$du = 5(-0.6)e^{-0.6t}\, dt$

$-\frac{1}{3}du = e^{-0.6t}\, dt$

$t=0: \ u = 1 + 5e^0 = 6$

$t=20: \ u = 1 + 5e^{-0.6(20)} = 1 + 5e^{-12} \approx 1.000030721$

b) $= 20000\, u^{-1}\Big|_6^{1.000030721}$

$= 20000\left[\frac{1}{1.000030721} - \frac{1}{6}\right] \approx \boxed{16{,}666.052\cancel{23}27}$

bob forgot this "2" when he typed in the upper limit, should have copied and pasted or left as $1 + 5e^{-12}$

c) Maple finds instead

$\Delta V_{maple} = 16666.052206$

$\Delta V = 16666.052\cancel{23}27$

~~differ by 23~~ differ in final digit

~~same!~~

If you truncate here you lose accuracy in result.

You say: "~~Pretty~~ close!" still valid!

bob says: ~~Keeping only 10 digits in the intermediate number for the limit of integration loses 10 digit accuracy in the result.~~ still valid! ~~If you increase Digits to 15, you get the same result to 10 digits.~~

~~not necessary, but could have been~~

ALWAYS keep max # digits in intermediate numbers