MAT1505-03/04 17F Final Exam Print Name (Last, First) ________________________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).p

1. The cochleoid described by the polar equation $r = \frac{\sin(\theta)}{\theta}$ spirals into the origin with an infinite series of loops.

a) Set up a fully simplified integral for the arclength L of the loop $-\pi \le \theta \le \pi$, and evaluate it numerically to 4 digit accuracy. This is an improper integral. Why? Plot its integrand over this interval to confirm that it is well behaved (continuous) on that interval and give a rough sketch, labeling axes and tickmarks.

[Optional: $L = \int_{-\pi}^{\pi} \sqrt{\frac{numer(\theta)}{denom(\theta)}}\, d\theta$. Expand $numer(t)$ in a Taylor series at $\theta = 0$ to evaluate $\lim_{\theta \to 0} \frac{numer(\theta)}{denom(\theta)}$; helpful: $> taylor(f(\theta), \theta = 0, 6)$].

b) Set up a definite integral for the area enclosed by the cochleoid loop $-\pi \le \theta \le \pi$, exactly and then to 4 significant digit accuracy.

c) Make a rough sketch of this curve segment together with the inscribed circle $r = \cos(\theta)$, and compare your previous results with the area and circumference of this circle. Are they reasonable? [Be sure to include tickmarks labeling your axes in your sketch. Remember the formulas for the circumference and area of a circle!]

d) Write down in simplest form the equation whose solution determines the angle $0 \le \theta \le \frac{\pi}{2}$ in the first quadrant where the tangent line is horizontal, and solve it numerically to 10 digit accuracy and evaluate the Cartesian coordinates of this point to 4 digit accuracy.

2. The right strophoid $x = \frac{1 - t^2}{1 + t^2}$, $y = \frac{t \cdot (t^2 - 1)}{1 + t^2}$ traces out the curve $y^2 = x^2 \frac{1 - x}{1 + x}$, with the graph $y = x\sqrt{\frac{1 - x}{1 + x}}$ corresponding to the one half of this curve.

a) Make a rough sketch of this curve for $-3 \le t \le 3$. Annotate the key points (endpoints and axis intercepts) you see with the values of (t, x, y) and put an arrowhead on each segment of the curve indicating increasing values of its parameter.

b) What is the limit of x as $t \to \pm\infty$? What does this tell you about the asymptotic behavior of the curve?

c) Evaluate the slopes of the tangent lines of the points on this parametrized curve which pass through the origin. What (smallest) angles $\Theta = \pm\Theta_0$, $\Theta_0 > 0$ do these slopes $m = \tan(\Theta)$ correspond to?

d) For what interval is the closed loop portion of this curve traced out? What interval corresponds to the upper half of the loop? Evaluate the exact area of this loop using its parametric representation (state the simplified integral you evaluate) and give its numerical value to 4 decimal places.

e) Evaluate the exact area of the loop by doubling the area of the upper half of the loop obtained using its equation as a graph of y versus x and give its numerical value to 4 decimal places.

f) Since $\frac{y}{x} = \tan(\theta)$, it is relatively straightforward to convert the Cartesian equation of this curve to the polar coordinate form

$$r = \frac{\cos^2(\theta) - \sin^2(\theta)}{\cos(\theta)} = \ldots = 2\cos(\theta) - \sec(\theta).$$

At what values of $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ does this pass through the origin?

Evaluate the area of the inner loop using this polar form of the curve and compare its exact value with the results of part d) and e).

f) **Optional.** Show that the parametrized curve satisfies the given Cartesian equation of this curve. Derive the polar form of the Cartesian form of this curve.

solution

Remember for a single parametrized curve, or a pair of polar coordinate curves:

> $plot([t, t^2, t = 0 .. 1], scaling = constrained)$

> $plot([1, \cos(\theta)], \theta = 0 .. \pi, color = [red, blue], coords = \text{polar}, scaling = constrained)$

Remember: " " (space) or "*" for multiplication ALWAYS, $\pi \neq$ pi, e $\neq e$.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: