MAT1505-03/04 17F Test 2 Print Name (Last, First) _____ , _____|____
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC). Note: all antiderivatives and derivatives may be performed with technology.

1. Find the arclength L of the curve segment $y = \sin(x),\ 0 \le x \le \pi$ exactly and to 3 decimal places. First state the simplified integral needed, and then use technology to evaluate it.

2. Use a limit to evaluate the following improper integral exactly and numerically to 5 decimal places: $\int_0^1 \frac{1}{\sqrt{x}\cdot(1+x)}\,dx$. (Use technology to get the antiderivative, although it is easily obtained by the variable substitution $u = \sqrt{x}$).

3. a) Show that $p(x) = \frac{6x}{(x+1)^4}$ is a probability distribution (called a beta prime distribution) on the interval $0 \le x < \infty$ by showing that its integral over this semi-infinite interval has the value 1. [Be sure to use limits, and show the integration steps here and below, relying on technology for the antiderivative you need, although it can easily be done by the variable substitution $u = x + 1$.]
b) What is the expected value μ of x ?
c) What is the probability that the random variable x assumes a value less than or equal to μ?

4. a) $E_{avg} = \frac{2\pi N}{(\pi k T)^{\frac{3}{2}}} \int_0^\infty E^{\frac{3}{2}}\, e^{-\frac{E}{kT}}\,dE$ is the average energy of a system of N gas atoms distributed in energies following the classical Maxwell–Boltzmann distribution. Re-express this integral in terms of the dimensionless energy variable $x = \frac{E}{kT}$ in order to determine the geometrical factor C in the result $E_{avg} = C\,N\,k\,T$, where C is a definite integral expressed entirely in terms of x only.
b) Now evaluate C exactly using technology to obtain a final formula for E_{avg}.

▶ solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: