MAT1505-03/04 17F Test 1 Answers

① a) $\langle v \rangle = \dfrac{\int_0^R \frac{P}{4\eta\ell}(R^2-r^2)\, 2\pi r\, dr}{\pi R^2}$

$x=1 \leftarrow r=R$

$= \dfrac{P}{4\eta\ell} \int_0^R \Big(1 - \underbrace{\tfrac{r^2}{R^2}}_{x^2}\Big)\, 2\, \underset{Rx}{r}\, \underset{R\,dx}{dr}$

$x=0 \leftarrow r=0$

$\left(\begin{array}{l} x = \frac{r}{R},\ dx = \frac{dr}{R} \\ r = Rx,\ dr = R\,dx \end{array}\right) \qquad v(0) = \dfrac{PR^2}{4\eta\ell} = v_{max}$

$= \underbrace{\dfrac{2PR^2}{4\eta\ell}}_{2v_{max}} \int_0^1 (1-x^2)\, x\, dx$

$= \boxed{2 v_{max} \int_0^1 (1-x^2)\, x\, dx}$

(b) $\overset{}{=}\ 2 v_{max} \underbrace{\int_0^1 x - x^3\, dx}_{\frac{x^2}{2} - \frac{x^4}{4}\big|_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}}$

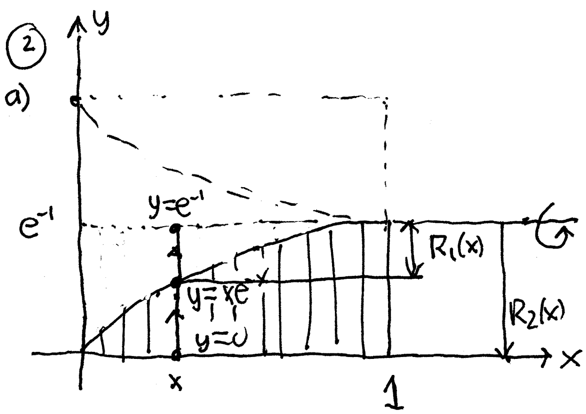
$= \boxed{\frac{1}{2} v_{max}}$

(c) $v = v_{max}\left(1 - \frac{r^2}{R^2}\right) = \frac{1}{2} v_{max}$

$1 - \frac{r^2}{R^2} = \frac{1}{2}$

$\frac{1}{2} = \frac{r^2}{R^2} \rightarrow \boxed{\frac{r}{R} = \frac{1}{\sqrt{2}}} \approx 0.7071$

② a)



$R_1(x) = e^{-1} - xe^{-x}$

$R_2(x) = e^{-1}$

$0 = \frac{dy}{dx} = \frac{d}{dx}(xe^{-x}) = 1e^{-x} + xe^{-x}(-1)$

$= e^{-x}(1-x) \rightarrow x = 1 \rightarrow y = 1e^{-1} = e^{-1}$ ✓

$V = \pi \int_0^1 R_2(x)^2 - R_1(x)^2\, dx \longrightarrow$

② a) continued

$V = \pi \int_0^1 \underbrace{(e^{-1})^2 - (e^{-1} - xe^{-x})^2}\, dx$

$\cancel{e^{-2}} - (\cancel{e^{-2}} - 2xe^{-1-x} + x^2e^{-2x})$

$= 2xe^{-1-x} - x^2e^{-2x}$

$= \boxed{\pi \int_0^1 2x\, e^{-1-x} - x^2 e^{-2x}\, dx}$

b) $= \boxed{\pi\left(-\frac{1}{4} + 2e^{-1} - \frac{11}{4}e^{-2}\right) \approx 0.3568}$

c) $V_{cyl} = \pi r^2 h = \pi(e^{-1})^2(1) = \pi e^{-2} \approx 0.4252$

$\frac{V}{V_{cyl}} \approx 0.8392 \rightsquigarrow \boxed{84\%}$

③ b) $\int_0^{\frac{\pi}{4}} \underbrace{t}_{u}\ \underbrace{\sin 2t\, dt}_{dv} = -\frac{t}{2}\cos 2t\Big|_0^{\frac{\pi}{4}} - \underbrace{\int_0^{\frac{\pi}{4}} -\frac{1}{2}\cos 2t\, dt}_{+\frac{1}{2}\int_0^{\frac{\pi}{4}} \cos 2t\, dt}$

$du = dt \leftarrow v = -\frac{1}{2}\cos 2t$

$\underbrace{+\frac{1}{2}\int_0^{\frac{\pi}{4}} \cos 2t\, dt}_{\frac{1}{4}\sin 2t\big|_0^{\frac{\pi}{4}}}\ \cancel{+C}$

$= \left(-\frac{t}{2}\cos 2t + \frac{1}{4}\sin 2t\right)\Big|_0^{\pi/4}$

$= -\frac{\pi}{4}\underbrace{\cos\frac{\pi}{2}}_{0} + \frac{1}{4}\underbrace{\sin\frac{\pi}{2}}_{1} - (0+0) = \boxed{\frac{1}{4}}$

a) $\int_0^{\pi/4} t \sin 2t\, dt \overset{\text{maple}}{=} \boxed{\frac{1}{4}}$