MAT2500-01/04 16S Quiz 3 Print Name (Last, First) ____________________|___

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given the vector-valued function $\vec{r}(t) = \langle \cos(t), \sin(t), \sin(2t) \rangle$, $0 \le t \le 2\pi$ (no credit for unidentified expressions in your responses):

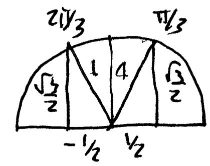
a) Evaluate $\vec{r}\,'(t)$, $\vec{r}\,''(t)$, $|\vec{r}\,'(t)|$, $\vec{T}(t)$ and remember to simplify your results.

b) Evaluate $\vec{r}\left(\frac{\pi}{3}\right)$, $\vec{r}\,'\left(\frac{\pi}{3}\right)$, $\vec{r}\,''\left(\frac{\pi}{3}\right)$, $\vec{T}\left(\frac{\pi}{3}\right)$ and remember to simplify your results.

c) Evaluate the exact angle θ in radians between $\vec{r}\,'\left(\frac{\pi}{3}\right)$ and $\vec{r}\,''\left(\frac{\pi}{3}\right)$ and a single decimal place approximation in degrees.

d) Evaluate the vector $\vec{w}$ which is the vector projection of $\vec{r}\,''\left(\frac{\pi}{3}\right)$ orthogonal (perpendicular!) to $\vec{r}\,'\left(\frac{\pi}{3}\right)$.

► **solution** suspend boxing for a), b)



a) $\vec{r} = \langle \cos t, \sin t, \sin 2t \rangle$

$\vec{r}\,' = \langle -\sin t, \cos t, 2\cos 2t \rangle$

$\vec{r}\,'' = \langle -\cos t, -\sin t, -4\sin 2t \rangle$

$|\vec{r}\,'| = \sqrt{\sin^2 t + \cos^2 t + 4\cos^2 2t}$

$= \sqrt{1 + 4\cos^2 2t}$

$\hat{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \frac{\langle -\sin t, \cos t, 2\cos 2t \rangle}{\sqrt{1+4\cos^2 2t}}$

b) $\vec{r}\left(\frac{\pi}{3}\right) = \langle \cos\frac{\pi}{3}, \sin\frac{\pi}{3}, \sin\frac{2\pi}{3} \rangle$

$= \langle \frac{1}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2} \rangle$

$\vec{r}\,'\left(\frac{\pi}{3}\right) = \langle -\sin\frac{\pi}{3}, \cos\frac{\pi}{3}, 2\cos\frac{2\pi}{3} \rangle$

$= \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, 2\left(-\frac{1}{2}\right) \rangle = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, -1 \rangle$

$\vec{a} \equiv \vec{r}\,''\left(\frac{\pi}{3}\right) = \langle -\cos\frac{\pi}{3}, -\sin\frac{\pi}{3}, -4\sin\frac{2\pi}{3} \rangle$

$= \langle -\frac{1}{2}, -\frac{\sqrt{3}}{2}, -4\left(\frac{\sqrt{3}}{2}\right) \rangle = \langle -\frac{1}{2}, -\frac{\sqrt{3}}{2}, -2\sqrt{3} \rangle$

for d)

$\hat{T}\left(\frac{\pi}{3}\right) = \frac{\vec{r}\,'\left(\frac{\pi}{3}\right)}{\sqrt{1+4\underbrace{\cos^2\frac{2\pi}{3}}_{1/4}}} = \frac{1}{\sqrt{2}} \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, -1 \rangle$

c) $\widehat{\vec{r}\,''}\left(\frac{\pi}{3}\right) = \frac{\langle -1, -\sqrt{3}, -4\sqrt{3} \rangle}{\sqrt{1+3+16\cdot 3}} = -\frac{\langle 1, \sqrt{3}, 4\sqrt{3} \rangle}{\sqrt{52}}$

$\cos\theta = \hat{T}\left(\frac{\pi}{3}\right) \cdot \widehat{\vec{r}\,''}\left(\frac{\pi}{3}\right) = \frac{1}{2\sqrt{2}} \langle -\sqrt{3}, 1, -2 \rangle \cdot \left(-\frac{1}{\sqrt{52}}\right) \langle 1, \sqrt{3}, 4\sqrt{3} \rangle = -\frac{1}{2\sqrt{2}\sqrt{52}} (\cancel{-\sqrt{3} + \sqrt{3}} - 8\sqrt{3})$ 0

$= \frac{1}{2\sqrt{2}\sqrt{52}} (8\sqrt{3}) = \frac{2\sqrt{2}\sqrt{3}}{\sqrt{4 \cdot 13}} = \sqrt{\frac{6}{13}}$

$\boxed{\theta = \arccos\sqrt{\frac{6}{13}} \approx 47.2^\circ}$

d) $\vec{a}_{\parallel} = \left(\hat{T}\left(\frac{\pi}{3}\right) \cdot \vec{r}\,''\left(\frac{\pi}{3}\right)\right) \hat{T}\left(\frac{\pi}{3}\right) = \frac{1}{2\sqrt{2}} \langle -\sqrt{3}, 1, -2 \rangle \cdot \left(\frac{-1}{2}\right) \langle 1, \sqrt{3}, 4\sqrt{3} \rangle \frac{1}{2\sqrt{2}} \langle -\sqrt{3}, 1, -2 \rangle$

$= -\frac{1}{16} (-\sqrt{3} + \sqrt{3} - 8\sqrt{3}) \langle -\sqrt{3}, 1, -2 \rangle = \frac{\sqrt{3}}{2} \langle -\sqrt{3}, 1, -2 \rangle = \langle -\frac{3}{2}, \frac{\sqrt{3}}{2}, -\sqrt{3} \rangle$

oops!

$\vec{a}_{\perp} = \vec{a} - \vec{a}_{\parallel} = \langle -\frac{1}{2}, -\frac{\sqrt{3}}{2}, -2\sqrt{3} \rangle - \langle -\frac{3}{2}, \frac{\sqrt{3}}{2}, -\sqrt{3} \rangle$

$= \boxed{\langle 1, -\sqrt{3}, -\sqrt{3} \rangle = \vec{w}}$