MAT2500-01/04 16S Final Exam   Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL signs and arrows when appropriate. Always SIMPLIFY expressions. LABEL parts of problem. BOX final short answers. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Use technology to evaluate any integrals you set up.

1. a) Find the cylindrical and spherical coordinates (exactly and numerically to several digits, angle to one decimal place) which describe the circular curve of intersection between two spheres, one a unit sphere at the origin and the other a sphere of radius 2 at (0,0,2). Support your detailed calculations by an *r-z* half plane diagram labeling the axes, tickmarks and all key points and lines.
b) Set up an integral in spherical coordinates to evaluate the volume in the region outside the smaller sphere but inside the larger sphere.
c) Evaluate this integral with technology.
d) What percent of the volume of the larger sphere does this represent?

2. Verify Green's Theorem $\int_C \vec{F} \cdot d\vec{r} = \iint_R \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \, dA$ for the vector field $\vec{F} = \langle -x^2 y, x y \rangle$ around the counterclockwise bounding curve $C = C_1 + C_2$ of the region R, where C_1 is the graph $y = 4 - x^2$ for $x = -1..1$ and C_2 is the graph $y = 3$ for $x = -1..1$.

3. $\vec{\boldsymbol{F}} = \langle y z, z x, x y \rangle$.
a) Evaluate $\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$ along the twisted cubic curve segment $\vec{\boldsymbol{r}} = \langle t, t^2, t^3 \rangle$, $t = -\frac{1}{2}..1$.
b) Show that $\vec{\boldsymbol{F}}$ satisfies the curlfree condition that it admit a potential function, i.e., is a conservative vector field.

c) Find a potential function f for it.

d) Use the potential to evaluate the line integral $\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$ over any curve between the same endpoints with the same orientation.

4. a) Deconstruct this double integral $\iint_R x y \, dA = \int_0^{\frac{\sqrt{3}}{2}} \int_{1-\sqrt{1-y^2}}^{\frac{y}{\sqrt{3}}} x y \, dx \, dy + \int_0^{\frac{1}{2}} \int_{\sqrt{2x - x^2}}^{\sqrt{1-x^2}} x y \, dy \, dx$, namely identify the two regions of integration R_1 and R_2 that fit together into a single region R, supporting your work first with a completely labeled diagram of the two regions of integration indicating their relationship to the corresponding integrals in the usual fashion, showing for each region equally spaced cross-sections that shade each region indicating one typical bullet endpoint cross-section line segment representing the inner integral (arrow in the middle indicating the direction of the integrating variable, endpoint values of that variable stated).
b) Now redo the diagram appropriate for a single simple integral over R in polar coordinates and re-express the integral in those coordinates.
c) Evaluate both integral expressions exactly using technology (no decimals in expression) and numerically to 5 decimal places to make sure they agree.

▶ solution (on-line) turn over to sign pledge!

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in with your answer sheets as a cover page, with the Lastname, FirstName side face up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: