Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Explain in as many words as possible everything you are doing! For each hand integration step, state the antiderivative formula used before substituting limits into it: $\int_a^b f(x)\ dx = F(x)|_{x=a}^{x=b} = F(b) - F(a)$. Every integral should be checked with Maple.

1. $\int_0^1 \int_{\sqrt{y}}^1 \dfrac{y\, e^{x^2}}{x^3}\, dx\, dy$.

a) Make a completely labeled (shaded by typical cross-sections) diagram of the region of integration for this integral, with a typical correctly labeled cross-section line segment (bullet endpoints, arrowhead) representing the current iteration of the integral.
b) Redo this diagram appropriate for the reversed order of integration and evaluate it by hand step by step.

2. $\int_0^1 \int_x^{\sqrt{2x-x^2}} x\, y\, dy\, dx$.

a) Make a completely labeled (shaded by typical cross-sections) diagram of the region of integration for this integral, with a typical correctly labeled cross-section line segment (bullet endpoints, arrowhead) representing the current iteration of the integral.
b) Make a new diagram appropriate for the evaluation of this integral in polar coordinates.
c) Convert the integral to polar coordinates and simplify.
d) Evaluate this integral by hand step by step.

3. Consider the solid region R in the first corresponding to the triple integral $\int_0^2 \int_{y^3}^8 \int_0^{y^2} x\, y\, z\, dz\, dx\, dy$

See the figure illustrating R on page 2.
a) Re-iterate this triple integral in the order $dx\, dy\, dz$, showing how you "deconstruct" it to understand the region R. Support your new limits of integration with a diagram for the outer double integral with completely labeled line segment cross-sections and equally spaced such cross-sections for the shading, and a 3d diagram for the innermost integral indicating one typical completely labeled linear cross-section.
b) Check your two integrals exactly using technology, reporting Maple's results. They should agree with each other. Do they?

4. Consider the solid region R inside the unit sphere centered at $(0,0,1)$ and outside the unit sphere at the origin.
a) Write equations for the corresponding two circles in the r-z half plane and find the values of (r, z) at their intersection point. What is the corresponding value of the spherical coordinate φ at this point?
b) Make an r-z half plane diagram illustrating these two spheres and "shade in" the region R by equally spaced radial line segments appropriate for spherical coordinate integration over this region, labeling a typical one by the starting and stopping value equations for the radial coordinate (bullet endpoints, arrowhead). Include line segments for the starting and stopping values of the spherical coordinate φ for integration over R, labeled by their equations.
c) Set up spherical coordinate integrals for the volume V and xy-moment $M_{xy} = \iiint z\, dV$ of R .
d) Evaluate these integrals step by step by hand exactly and then numerically approximate them and their exact ratio $\bar{z}$ to 2 decimal places.
e) Mark your centroid point on one of your diagrams, identifying it. Does this value of the centroid height seem reasonable? Explain its value in relation to the plane $z = 1$.

5.a) Use cylindrical coordinates to re-express the triple integral $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} \sqrt{x^2+y^2}\ dz\ dy\ dx$,

supporting your limits of integration with labeled diagrams representing the outer double integral and the innermost integral.

b) Evaluate this new integral exactly by hand step by step and numerically to 4 significant figures. Does this agree with the numerical value of the original Cartesian integral?

Problem 3 integral region illustrated from two perspectives:
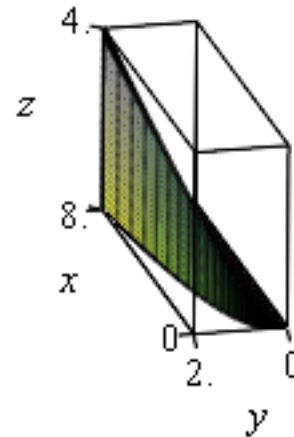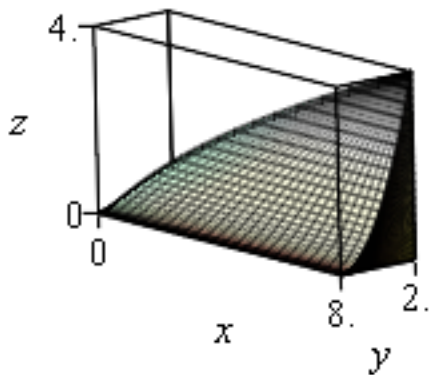
▶ solution (on-line)

No collaboration. You may only talk to bob. See test rules <u>on-line</u>. Read short rules above. Print out and attach any Maple supporting work you do, hand annotating if necessary with problem number and part etc, whatever is necessary for clarification.

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in stapled to your answer sheets as the cover page, with the Lastname, FirstName side face up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: