MAT2500-01/04 16S Test 2 Answers

① $f(x,y,z) = x\sqrt{y^2+z^2} = x\,(y^2+z^2)^{1/2}$

a) $f(2,3,4) = 2\sqrt{3^2+4^2} = 2(5) = 10$

level surface: $\boxed{x\sqrt{y^2+z^2} = 10}$

b) $\vec{\nabla} f(x,y,z) = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$

$= \langle (y^2+z^2)^{1/2}, x(\tfrac{1}{2})(y^2+z^2)^{-1/2}(2y), x(\tfrac{1}{2})(y^2+z^2)^{-1/2}(2z) \rangle$

$= \langle (y^2+z^2)^{1/2}, \frac{xy}{(y^2+z^2)^{1/2}}, \frac{xz}{(y^2+z^2)^{1/2}} \rangle$

$\vec{\nabla} f(2,3,4) = \langle 5, \frac{2\cdot 3}{5}, \frac{2\cdot 4}{5} \rangle = \langle 5, \frac{6}{5}, \frac{8}{5} \rangle$

$= \frac{1}{5}\langle 25,6,8 \rangle$

$\hat{u} = \widehat{\nabla f}(2,3,4) = \frac{\langle 25,6,8\rangle}{\sqrt{25^2+6^2+8^2}} \overset{\text{Maple}}{=} \boxed{\frac{\langle 25,6,8\rangle}{5\sqrt{29}}}$

$D_{\hat{u}} f(2,3,4) = |\vec{\nabla} f(2,3,4)| = \frac{1}{5}\sqrt{25^2+6^2+8^2} = \frac{1}{5}5\sqrt{29}$

$= \boxed{\sqrt{29}} \approx 5.385$

c) $\vec{v} = \langle 2,3,4 \rangle$, $|\vec{v}| = \sqrt{2^2+3^2+4^2} = \sqrt{4+9+16} = \sqrt{29}$

$\hat{v} = (1/\sqrt{29})\langle 2,3,4\rangle$

$D_{\hat{v}} f(2,3,4) = \hat{v}\cdot\vec{\nabla} f(2,3,4)$

$= \frac{1}{\sqrt{29}}\langle 2,3,4\rangle \cdot \frac{1}{5}\langle 25,6,8\rangle = \frac{1}{5\sqrt{29}}(50+18+32)$

$= \frac{100}{5\sqrt{29}} = \boxed{\frac{20}{\sqrt{29}}} \approx 3.714$

(omitted to shorten test)

~~d)~~ $\vec{r}_0 = \langle 2,3,4\rangle$, $\vec{n} = \langle 25,6,8 \rangle$ $(\propto \vec{\nabla} f(2,3,4))$

$0 = \vec{n}\cdot(\vec{r}-\vec{r}_0) = \langle 25,6,8\rangle \cdot \langle x-2, y-3, z-4\rangle$

$= 25(x-2) + 6(y-3) + 8(z-4)$

$= 25x + 6y + 8z \underbrace{-50-18-32}_{-100}$

$\boxed{25x+6y+8z = 100}$

e) $f(1.98, 3.05, 3.95) = 1.98\sqrt{3.05^2+3.95^2}$

(Maple) ≈ 9.881172

$L(x,y,z) = f(2,3,4) + \vec{\nabla} f(2,3,4)\cdot(\langle x,y,z\rangle - \langle 2,3,4\rangle)$

$= 10 + \frac{1}{5}\langle 25,6,8\rangle \cdot \langle x-2, y-3, z-4\rangle$

$= 10 + \frac{1}{5}(25(x-2) + 6(y-3) + 8(z-4))$

$L(1.98, 3.05, 3.95) = 10 + \frac{1}{5}(25(1.98-2) + 6(3.05-3) + 8(3.95-4))$

$= 10 + \frac{1}{5}(25(-0.02) + 6(0.05) + 8(-0.05))$

$= 10 + \frac{1}{5}(-0.5 + 0.3 - 0.4) = 10 - \frac{0.6}{5} = \boxed{9.88} \approx f(1.98, 3.05, 3.95)$

pretty close

② $S = 0.1091\, w^{0.425} h^{0.725}$

$dS = \frac{\partial S}{\partial w}dw + \frac{\partial S}{\partial h}dh$

$= 0.1091\,[0.425\, w^{0.425-1} h^{0.725} dw + 0.725\, w^{0.425} h^{0.725-1} dh]$

$\frac{dS}{S} = \frac{0.1091\,[\frac{0.425}{w} w^{0.425} h^{0.725} dw + \frac{0.725}{h} w^{0.425} h^{0.725} dh]}{0.1091\, w^{0.425} h^{0.725}}$

$= \frac{0.425\,dw}{w} + \frac{0.725\,dh}{h}$

$\left|\frac{dS}{S}\right| \le 0.425 \underbrace{\left|\frac{dw}{w}\right|}_{\le .02} + 0.725 \underbrace{\left|\frac{dh}{h}\right|}_{\le 0.02}$

$\le (0.425 + 0.725)(0.02) = 0.023$

so maximum calculated error $\le \boxed{2.3\%}$

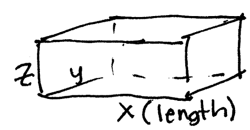
③ $f(x,y) = x^2 - xy + y^2 + 9x - 6y + 10$

$f_x = 2x - y + 9 = 0$
$f_y = -x + 2y - 6 = 0$
solve Maple: $(x,y) = (-4,1)$

$f_{xx} = 2 > 0$
$f_{yy} = 2 > 0$
local min? yes!

$f_{xy} = -1$ $\quad f_{xx}f_{yy} - f_{xy}^2 = 4 - (-1)^2 > 0$ ✓

④



constraint: $x + 2(y+z) = 9$

eliminate x: $x = 9 - 2(y+z) > 0$

Maximize: $V = xyz$

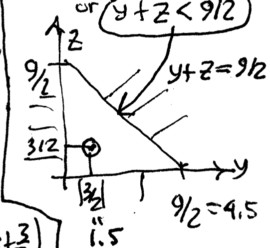
$= yz(9 - 2(y+z))$

$V = 9yz - 2y^2z - 2yz^2$ for $y>0, z>0$,

$9 - 2(y+z) > 0$ or $y+z < 9/2$

$\frac{\partial V}{\partial y} = 9z - 4yz - 2z^2 = z(9-4y-2z) = 0$

$\frac{\partial V}{\partial z} = 9y - 2y^2 - 4yz = y(9-2y-4z) = 0$



solve: $4y + 2z = 9 \rightarrow 6z = 9$

$2y + 4z = 9$ $\quad z = 3/2 = y$

$-: 2y - 2z = 0 \rightarrow y = z$

$x = 9 - 2(\frac{3}{2} + \frac{3}{2})$

$= 3$

so girth dimensions are 1.5 ft and length 3 ft for the largest such box, with volume $3(\frac{3}{2})(\frac{3}{2})$

$= \frac{27}{4} = 6.75\,ft^3$