

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). You are encouraged to use technology to check all of your hand results, but **everything on this test is straightforward to evaluate by hand and must be shown explicitly** (you may use Maple to solve pairs of equations for critical points).

1.  $f(x, y, z) = x\sqrt{y^2 + z^2}$ ;  $P(2, 3, 4)$

a) Write an equation for the level surface through  $P$ .

b) Find a unit vector in the direction in which this function increases most rapidly at  $P$ . What is its rate of change in this direction?

c) Evaluate the directional derivative of  $f$  at  $P$  in the direction away from the origin.

d) Numerically evaluate  $f(1.98, 3.05, 3.95)$  and then use the linear approximation at  $P$  to approximate this value.

2. A model for the surface area (in square feet) of a human body is  $S = 0.1091 w^{0.425} h^{0.725}$ , where  $w$  is the weight in pounds and  $h$  is the height in inches. If the errors in measurement in  $w$  and  $h$  are at most 2 percent, use differentials to estimate the maximum percentage error in the calculated surface area.

3.  $f(x, y) = x^2 - xy + y^2 + 9x - 6y + 10$ .

Find the critical points of  $f$  and use the second derivative test to classify these critical points as local minima, local maxima or saddle points. Use words to explain your reasoning. [Does an appropriate 3d or contour plot confirm your conclusion?] You may use Maple to solve for the critical points.

4. The sum of the length and girth (perimeter of the cross-section with the two smaller dimensions) of a rectangular box must be at most 9 feet to be mailed with the US Postal Service. Find the dimensions and volume (in cubic feet) of the largest box that meets this criterion. State clearly the domain of the function of 2 variables over which you are looking for your critical point and make a sketch of this domain including the solution point, labeling the edges and vertices of the region appropriately. [You may use Maple to solve for the critical point.]

## ► solution

## ▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:

"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: \_\_\_\_\_

Date: \_\_\_\_\_