MAT2705-01/02 16F Quiz 4   Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $4\,x_1 + 8\,x_2 + 3\,x_3 = -2$

$x_1 + 2\,x_2 - 3\,x_4 = 1$

$2\,x_1 + 4\,x_2 + x_3 - 2\,x_4 = 0$

a) Write down the coefficient matrix **A**, the RHS matrix $\vec{\mathbf{b}}$ and the augmented matrix $\mathbf{C} = \langle \mathbf{A} \mid \vec{\mathbf{b}} \rangle$ for this linear system of equations.

b) With technology (identify your choice!), reduce this matrix **C** step by step to its ReducedRowEchelonForm avoiding fractions ~~(7 steps!)~~, recording the intermediate matrices and row operations for each step (as in $R_1 \leftrightarrow R_2$, $R_3 \to R_3 + 2\,R_1$, $R_1 \to \frac{1}{2}\,R_1$). You may combine the AddRow operations within a single pivot, reporting only the final matrix.

c) Write out the equations that correspond to the reduced matrix. Identify the leading variables and the free variables and solve. State your solution in the scalar form: $x_1 = \ldots, x_2 = \ldots$, etc .

d) Enter the augmented matrix into Maple and by right clicking, find the reduced matrix and the solution of the system of equations. Write down exactly what Maple gives you for the column matrix solution and compare with your reduced matrix and solution. They should agree. Do they?

▶ solution

① $4x_1 + 8x_2 + 3x_3 + 0x_4 = -2$

$1x_1 + 2x_2 + 0x_3 - 3x_4 = 1$

$2x_1 + 4x_2 + 1x_3 - 2x_4 = 0$

$$A = \begin{bmatrix} 4 & 8 & 3 & 0 \\ 1 & 2 & 0 & -3 \\ 2 & 4 & 1 & -2 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

(or $R_2 \leftrightarrow R_3$)

$$C = \begin{bmatrix} 4 & 8 & 3 & 0 & -2 \\ 1 & 2 & 0 & -3 & 1 \\ 2 & 4 & 1 & -2 & 0 \end{bmatrix} \xrightarrow[b)]{R_1 \leftrightarrow R_2} \begin{bmatrix} \boxed{1} & 2 & 0 & -3 & 1 \\ 4 & 8 & 3 & 0 & -2 \\ 2 & 4 & 1 & -2 & 0 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - 4R_1} \begin{bmatrix} 1 & 2 & 0 & -3 & 1 \\ 0 & 0 & 3 & 12 & -6 \\ 0 & 0 & 1 & 4 & -2 \end{bmatrix} \xrightarrow{R_2 \to \frac{1}{3}R_2} \begin{bmatrix} 1 & 2 & 0 & -3 & 1 \\ 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 1 & 4 & -2 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - R_2} \begin{bmatrix} 1 & 2 & 0 & -3 & 1 \\ 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

easy to do by hand but the LinearSolveTutor in Maple is foolproof.

c) $\begin{matrix} x_1 & x_2 & x_3 & x_4 \\ L & F & L & F \end{matrix}$ —— $\boxed{\begin{matrix} \text{Leading vars}: & \{x_1, x_3\} \\ \text{Free vars}: & \{x_2, x_4\} \end{matrix}}$

c) reduced equations: $\boxed{\begin{matrix} x_1 + 2x_2 - 3x_4 = 1 \\ x_3 + 4x_4 = -2 \\ 0 = 0 \end{matrix}}$ $\to$ $x_1 = 1 - 2x_2 + 3x_4 = 1 - 2t_1 + 3t_2$

$x_3 = 2 - 4x_4 = 2 - 4t_2$

$x_2 = t_1$

$x_4 = t_2$

soln: $\boxed{x_1 = 1 - 2t_1 + 3t_2,\ x_2 = t_1,\ x_3 = 2 - 4t_2,\ x_4 = t_2}$ $\longrightarrow$ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 - 2t_1 + 3t_2 \\ t_1 \\ 2 - 4t_2 \\ t_2 \end{bmatrix}$

d) Maple output:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 - 2\_t_2 + 3\_t_2 \\ \_t_2 \\ -2 - 4\_t_4 \\ \_t_4 \end{bmatrix}$$

It looks like Maple uses the index of the free variable to label the corresponding parameters:

$t_1 = \_t_2$

$t_2 = \_t_4$

they agree with this correspondence.