MAT2705-02/05 15S Test 2 Print Name (Last, First) ____________________________________|__

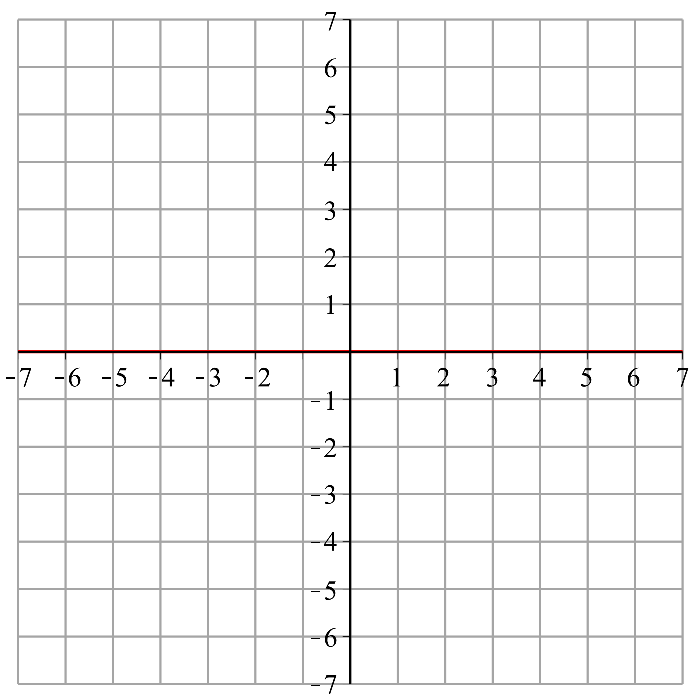
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **You may use technology for row reductions, determinants and matrix inverses.**

1. a) On the grid below, **draw in** arrows representing the vectors $\vec{v}_1 = \langle 1, 1\rangle$ and $\vec{v}_2 = \langle 3,-1\rangle$ and $\vec{v}_3 = \langle 3,-5\rangle$ and **label** them by their symbols. **Extend** the basis vectors $\{\vec{v}_1, \vec{v}_2\}$ to the corresponding coordinate axes for (y_1, y_2) and **mark** the positive direction with an arrow head and the axis label. Mark off tickmarks on these axes for integer values of the new coordinates.Then **draw in** the parallelogram that graphically expresses $\vec{v}_3$ as a linear combination of $\{\vec{v}_1, \vec{v}_2\}$. **Label** its two sides along the new coordinate axes by the corresponding vectors they represent. Then read off the coordinates (y_1, y_2) of $\vec{v}_3$ with respect to these two vectors (write them down) and **express** $\vec{v}_3$ as a linear combination of these vectors; **put this equation** at the tip of this vector.

b) Now write down the matrix equation that enables you to express $\vec{v}_3$ as a linear combination of the other two vectors, solve that system using the inverse coefficient matrix (stating its value and showing the matrix multiplication and simplification steps), and then express $\vec{v}_3$ explicitly as a linear combination of those vectors, writing $\vec{v}_3 =$...

c) Check your linear combination by expanding it out to get the original vector $\vec{v}_3$. Does your matrix result from part b) agree with the graphical result from part a)?

d) Now using the new coordinate axes, **draw in** the arrow representing the vector $\vec{\boldsymbol{v}}_4$ whose new coordinates are $(y_1, y_2) = (2,-2)$ and **label** the tip of $\vec{\boldsymbol{v}}_4$ by its symbol. Then draw in the projection parallelogram associated with the new coordinates for which $\vec{\boldsymbol{v}}_4$ is the main diagonal. Read off its old coordinates (x_1, x_2) from the grid (write them down). Do they agree with the linear combination $y_1 \vec{v}_1 + y_2 \vec{v}_2$?



2. Consider the following system of equations:

$$3x_1 + x_2 + x_3 + 6x_4 = 14$$
$$x_1 - 2x_2 + 5x_3 - 5x_4 = -7$$
$$4x_1 + x_2 + 2x_3 + 7x_4 = 17$$

a) Write this system in matrix form $A\vec{\boldsymbol{x}} = \vec{\boldsymbol{b}}$, identifying the coefficient matrix and the RHS matrix.
b) Find its general solution (writing down the augmented matrix and its RREF form, identifying Leading and Free variables, reduced equations, etc), stating the result in the vector form $\vec{\boldsymbol{x}} = \ldots$
c) Separate this into two terms $\vec{\boldsymbol{x}} = \vec{\boldsymbol{x}}_{part} + \vec{\boldsymbol{x}}_{hom}$, identifying the particular solution and the general solution of the related homogeneous matrix equation.
d) Let $A = \langle \vec{v}_1 | \vec{v}_2 | \vec{v}_3 | \vec{v}_4 \rangle$ define the columns of the coefficient matrix as vectors. Use the particular solution to express the vector $\vec{\boldsymbol{b}}$ as a unique linear combination of these vectors: $\vec{\boldsymbol{b}} = \ldots$
e) From the homogeous solution, identify a basis $\{\vec{u}_1, \ldots\}$ of the solution space of the homogeneous matrix equation.
f) Write down the independent linear relationships among the four vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$.

3. Consider the set of the following vectors:
$\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\} = \{\langle 1, 0, 2, 3\rangle, \langle -2, 3, -3, -2\rangle, \langle 3, 2, 4, 9\rangle, \langle 4, 1, 0, 5\rangle\}$
a) What does the value of the determinant of the matrix $A = \langle \vec{v}_1 | \vec{v}_2 | \vec{v}_3 | \vec{v}_4 \rangle$ say about the linear independence of this set?
b) What does the row reduced matrix obtained from A tell you about the dimension of the subspace that these vectors span? Explain. Give a basis of this subspace.

Sign and date the pledge at the end of your exam.

▶ solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: